Textbook: Larson, Hosterler and Edwards, Calculus, 7th edition, Houghton Mifflin, Boston, 2002. (2003-01-03)

Solutions to exercises in Section 5.8: #3, #9, #10, #11, #12, #17, #18, #53, #57, #75, #79, #80, #81, #82.

#3. False. The range of arccos is $[0, \pi]$. In fact, $\arccos \frac{1}{2} = \frac{\pi}{3}$.

#9. $\arctan \frac{\sqrt{3}}{3} = \frac{\pi}{6}$.

#10. $\operatorname{arccot}(-\sqrt{3}) = \frac{5\pi}{6}$.

#11. $\operatorname{arccsc}(-\sqrt{2}) = -\frac{\pi}{4}$.

#12. $\arccos\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$.

#17. (a) Let $x = \arctan \frac{3}{4}$, where $0 < x < \pi/2$. Then $\tan x = 3/4$ and so $\sec x = \sqrt{1 + (3/4)^2} = 5/4$. Thus $\sin\left(\arctan \frac{3}{4}\right) = \sin x = \tan x / \sec x = 3/5$.

(b) Let $x = \arcsin \frac{4}{5}$, where $0 < x < \pi/2$. Then $\sin x = 4/5$ and so $\cos x = \sqrt{1 - (4/5)^2} = 3/5$. Thus $\sec\left(\arcsin \frac{4}{5}\right) = \sec x = 1/\cos x = 5/3$.

#18. (a) Let $x = \arccos \frac{\sqrt{2}}{2}$, where $0 < x < \pi/2$. Then $\cos x = \sqrt{2}/2$ and so $\sin x = \sqrt{1 - (\sqrt{2}/2)^2} = \sqrt{2}/2$. Thus $\tan\left(\arccos \frac{\sqrt{2}}{2}\right) = \tan x = \sin x / \cos x = 1$.

(b) Let $x = \arcsin \frac{5}{13}$, where $0 < x < \pi/2$. Then $\sin x = 5/13$ and so $\cos x = \sqrt{1 - (5/13)^2} = 12/13$. Thus $\cos\left(\arcsin \frac{5}{13}\right) = \cos x = 12/13$.

#53. Since $y = \frac{1}{2}\left(\frac{1}{2} \ln \frac{x+1}{x-1} + \tan x\right) = \frac{1}{4}(\ln(x+1) - \ln(x-1) + 2\tan x)$, we have

$$y' = \frac{1}{4}\left(\frac{1}{x+1} - \frac{1}{x-1} + \frac{2}{x^2+1}\right) = \frac{1}{1-x^4}.$$

#57. Since $y = 8 \arcsin \frac{x}{4} - \frac{x\sqrt{16-x^2}}{2}$, we have

$$\begin{aligned} y' &= \frac{8(1/4)}{\sqrt{1-(x/4)^2}} - \frac{\sqrt{16-x^2}}{2} - \frac{x}{4}(16-x^2)^{-1/2}(-2x) \\ &= \frac{8}{\sqrt{16-x^2}} - \frac{\sqrt{16-x^2}}{2} + \frac{x^2}{2\sqrt{16-x^2}} \\ &= \frac{16-(16-x^2)+x^2}{2\sqrt{16-x^2}} \\ &= \frac{x^2}{\sqrt{16-x^2}}. \end{aligned}$$

#75. Let $\arctan x = a$ and $\arctan y = b$. Then, $\tan a = x$ and $\tan b = y$. And so,

$$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b} = \frac{x+y}{1-xy}.$$

Thus

$$\arctan x + \arctan y = a + b = \arctan \frac{x+y}{1-xy}.$$

Consequently,

$$\arctan \frac{1}{2} + \arctan \frac{1}{3} = \arctan \frac{1/2+1/3}{1-(1/2)(1/3)} = \arctan 1 = \frac{\pi}{4}.$$

#79. True. Notice that $\frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}$ is positive for all x.

#80. False. The range of $y = \arcsin x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

#81. True. Notice that $\frac{d}{dx}[\arctan(\tan x)] = \frac{d}{dx}[x] = 1$ for all x in the domain.

#82. False. For instance, $\arcsin^2 0 + \arccos^2 0 = 0 + (\pi/2)^2 \neq 1$.