The 5th International Conference
by Graduate School of Mathematics,
Nagoya University

第5回 名古屋国際数学コンファレンス

Geometric Quantization and Related Complex Geometry

November 16–19, 2005

Noyori Conference Hall,
Nagoya University, Nagoya, Japan

Invited Speakers

R. Bielawski (Edinburgh U.)
M. Bordemann (U. of Haute Alsace)
L. Charles (Paris 6 U.)
X.-X. Chen (U. of Wisconsin)
A. Futaki (Tokyo Tech)
P. Heinzner (Bochum U.)
R. Kobayashi (Nagoya U.)
T. Mabuchi (Osaka U.)
T. Moriyama (Osaka U.)
J. Rawnsley (Warwick U.)
A. Sergeev (Steklov Math. Inst.)
K. Sugiyama (Chiba U.)
H. Upmeier (U. of Marburg)
C.-L. Wang (National Central U.)
W. Zhang (Nankai U.)

Organizers

Akito Futaki (Tokyo Tech)
Ryoichi Kobayashi (Nagoya U.)
Yoshiaki Maeda (Keio U.)
Armen Sergeev (Steklov Math. Inst.)
Tohru Uzawa (Nagoya U.)

The 5th International Conference
by Graduate School of Mathematics, Nagoya University

Geometric Quantization and Related Complex Geometry

date: **November 16–19, 2005**
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PROGRAM

Wednesday, November 16, 2005

9:25–9:30 **Opening Address**

9:30–10:30 X.-X. Chen (Univ. of Wisconsin, Madison)
On the lower bound of Calabi energy

11:00–12:00 W. Zhang (Nankai Univ.)
Two themes in geometric quantization

13:30–14:30 K. Sugiyama (Chiba Univ.)
On a geometric non-abelian class field theory and an application to threefolds

15:00–16:00 P. Heinzner (Bochum Univ.)
Semistable points with respect to real forms

Thursday, November 17, 2005

9:30–10:30 T. Moriyama (Osaka Univ.)
Pre-symplectic geometry and the construction of pre-symplectic submanifolds

11:00–12:00 L. Charles (Paris 6 Univ.)
On the semi-classical naturality of quantization with half-form

13:30–14:30 R. Kobayashi (Nagoya Univ.)
Toward Nevanlinna/Galois theory of the Gauss map of pseudo-algebraic minimal surfaces

15:00–16:00 R. Bielawski (Edinburgh Univ.)
Asymptotic monopole metrics

Friday, November 18, 2005

9:30–10:30 A. Futaki (Tokyo Inst. of Tech.)
Harmonic total Chern forms and stability

11:00–12:00 C.-L. Wang (National Central Univ.)
Green functions on tori and the mean field equations

13:30–14:30 J. Rawnsley (Warwick Univ.)
Natural star products

15:00–16:00 H. Upmeier (Univ. of Marburg)
Quantization of Symmetric Spaces, including Super-Symmetry

Saturday, November 19, 2005

9:30–10:30 M. Bordemann (Univ. of Haute Alsace)
Quantization of coisotropic submanifolds

11:00–12:00 T. Mabuchi (Osaka Univ.)
Stability on polarized algebraic manifolds

13:30–14:30 A. Sergeev (Steklov Math. Inst., Moskow)
Seiberg-Witten equations on the non-commutative Euclidean 4-space

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ABSTRACTS

Nov. 16th (WED), 9:30–10:30

•**X.-X. Chen** (University of Wisconsin, Madison)

"On the lower bound of Calabi energy"

It is known before that the Calabi energy has sharp lower bound when the Kähler class admit an extremal metric. In this talk, we will give a lower bound for Calabi energy when the underlying complex structure is de-stablized by another complex structure.

Nov. 16th (WED), 11:00–12:00

•**W. Zhang** (Nankai University)

"Two themes in geometric quantization"

I would like to survey my two works related to geometric quantization and symplectic reduction. The first is my joint work with Youliang Tian on the analytic approach of the Guillemin-Sternberg geometric quantization conjecture. The other is a recent joint work with Xiaonan Ma on the asymptotic expansion of the invariant Bergman kernel on a symplectic manifold admitting with a Hamiltonnina group action of a compact connected Lie group.

Nov. 16th (WED), 13:30–14:30

•**K. Sugiyama** (Chiba University)

"On a geometric non-abelian class field theory and an application to threefolds"

Let K be a number field. Let $\mathcal{C}l_K$ be its idele class group and $\mathcal{C}l_K^{\circ}$ its identity component. Then the classical class field theory of the number theory tells us that there is an isomorphism between the group of connected components $\pi_0(\mathcal{C}l_K) = \mathcal{C}l_K/\mathcal{C}l_K^o$ of $\mathcal{C}l_K$ and the Galois group $\mathrm{Gal}(K^{ab}/K)$ where K^{ab} is the maximal abelian extension of K. This has the following geometric analogue. Let S be a compact Riemmanian surface. Then a local system χ over S naturally defines a local system of rank one $\check{\chi}$ on its Jacobian $Jac(S)$ and vice versa. We will call this correspondence as "a geometric abelian class field theory". We will discuss an extension of such a correspondence to a *non-abelian case*. More precisely we will discuss a correspondence between flat $PSL_2(\mathbb{C})$ bundles over S which has parabolic reductions at certain points $\{P_i\}$ and perverse sheaves over the modular stack of principal $SL_2(\mathbb{C})$ bundles over S which have paradolic reduction at $\{P_i\}$. Using a relative version of our geometric non-abelian class field theory, we will also discuss a relation between a characteristic polynomial of a monodromy representation of the KZ-connection and the Alexander polynomial of a certain threefold.

Nov. 16th (WED), 15:00–16:00

•**P. Heinzner** (Bochum University)

"Semistable points with respect to real forms"

In the context of classical geometric invariant theory in the sense of Mumford, the set of semistable points with respect to an action of a complex reductive group R on a complex space Z is defined in terms of ample line bundles. In this setting we may choose a maximal compact subgroup U of R, i.e., regard R as the complexification $U^{\mathbb{C}}$ of U, an U-invariant Káhlerian structure ω on Z and an U-equivariant moment map $\mu\colon Z \to \mathfrak{u}^*$, where $\mathfrak{u}$ denotes the Lie algebra of U and $\mathfrak{u}^*$ its dual. The set of semistable points is then given by $S_{U^{\mathbb{C}}}(\mu^{-1}(0)) = \{z \in Z|\ \overline{U^{\mathbb{C}}z} \cap \mu^{-1}(0) \neq \emptyset\}$.
In this talk we generalize the notion of semistability for actions of real forms G of $U^{\mathbb{C}}$. Here one may assume that the Lie algebra $\mathfrak{g}$ of G has a Cartan decomposition $\mathfrak{g} = \mathfrak{k} \oplus \mathfrak{p}$, where $\mathfrak{k} = \mathfrak{u} \cap \mathfrak{g}$ and $\mathfrak{p} = i\mathfrak{u} \cap \mathfrak{g}$. The inclusion $i\mathfrak{p} \overset{\iota}{\hookrightarrow} \mathfrak{u}$ leads to a map $\mu_{i\mathfrak{p}}\colon Z \to (i\mathfrak{p})^*$, $\mu_{i\mathfrak{p}} = \iota^* \circ \mu$ and by definition $S_G(\mu_{i\mathfrak{p}}^{-1}(0)) := \{z \in Z|\ \overline{Gz} \cap \mu_{i\mathfrak{p}}^{-1}(0) \neq \emptyset\}$ is the set of semistable points with regard to G.
We show that $S_G(\mu_{i\mathfrak{p}}^{-1}(0))$ is the right analog of $S_{U^{\mathbb{C}}}(\mu^{-1}(0))$ for real forms G of $U^{\mathbb{C}}$. In particular we show:

- $S_G(\mu_{i\mathfrak{p}}^{-1}(0))$ is open in Z.
- There exists a quotient $S_G(\mu_{i\mathfrak{p}}^{-1}(0))/\!\!/G$ which parametrizes the closed G-orbits in $S_G(\mu_{i\mathfrak{p}}^{-1}(0))$.

Moreover, the inclusion $\mu_{i\mathfrak{p}}^{-1}(0) \hookrightarrow S_G(\mu_{i\mathfrak{p}}^{-1}(0))$ induces a homeomorphism $\mu_{i\mathfrak{p}}^{-1}(0)/K \to S_G(\mu_{i\mathfrak{p}}^{-1}(0))/\!\!/G$.

Nov. 17th (THU), 9:30–10:30

•**T. Moriyama** (Osaka University)

"Pre-symplectic geometry and the construction of pre-symplectic submanifolds"

1. Introduction

Let ω be a symplectic form on a compact symplectic manifold with the integral class $[\frac{\omega}{2\pi}]$ in $H^2(M;\mathbb{Z})$. Then Donaldson shows that for a sufficiently large integer k the Poincaré dual of $[\frac{k\omega}{2\pi}]$ can be realised by a symplectic submanifold N_k which is the zero set of a section of a complex line bundle [2]. Later Auroux constructed a family of symplectic submanifolds obtained as zero sets of sections of a complex vecter bundle and proved that these submanifolds are isotopic [1]. Ibort-Martínez-Presas provided an analogy in contact geometry to these results [4]. Moreover, Donaldson constructed a Lefschetz pencil on a symplectic manifold by considering a pair of sections of a line bundle [3].

In this paper we introduce pre-symplectic manifolds as a generalization of symplectic manifolds. We provide a unified generalization of Donaldson's result to pre-symplectic manifolds, which includes Auroux's results, the analogy in contact geometry to Donaldson's result and Lefschetz's hyperplane theorem. Finally we construct Lefschetz type pencils on pre-symplectic manifolds of constant rank.

2. Pre-symplectic manifolds

Suppose that M is a C^∞-manifold of dimension $2n+\ell$, $(n>0, \ell \geq 0)$.

Definition 1 A closed 2-form ω on M is a *pre-symplectic form of height* n if $\omega_x^n \neq 0$ for all x $(\in M)$. A pair (M,ω) is a *pre-symplectic manifold of height* n. If $\omega_x^n \neq 0$ and $\omega_x^{n+1} = 0$ for all x $(\in M)$, ω is a *pre-symplectic form of rank* n and (M,ω) is a *pre-symplectic manifold of rank* n.

For a pre-symplectic $(2n+\ell)$-dimensional manifold M of height n, a $(2m+\ell)$-dimensional submanifold N of M is a *pre-symplectic submanifold of height* m *(resp. rank* m*)* if $(N, \omega|_N)$ is pre-symplectic manifold of height m (resp. rank m), $(m>0)$.

From now on, we suppose manifolds to be compact and oriented.

Theorem 1 *Let* (M,ω) *be a pre-symplectic* $(2n+\ell)$*-dimensional manifold of rank* n*. If the class* $[\frac{\omega}{2\pi}]$ *is integral, then for a sufficiently large integer* k *the Poincaré dual of* $[\frac{k\omega}{2\pi}]$*, in* $H_{2(n-1)+\ell}(M;\mathbb{Z})$*, can be realised by a pre-symplectic submanifold* N_k *of height* $n-1$*.*

Corollary 1 *If* $\ell = 0, 1$ *in Theorem 1, the submanifold* N_k *is of rank* $n-1$*.*

The case $\ell = 0$ implies the Donaldson's result.

3. Further results

Moreover, Theorem 1 can be extended as follows :

Theorem 2 *Let* (M, ω) *be a pre-symplectic* $(2n+\ell)$*-dimensional manifold of rank* n *and* E *a complex vector bundle of rank* r *over* M*,* $(r \leq n)$*. If the class* $[\frac{\omega}{2\pi}]$ *is integral, then for a sufficiently large integer* k *the Poincaré dual of* $[\frac{k\omega}{2\pi}]^r + c_1(E) \cdot [\frac{k\omega}{2\pi}]^{r-1} + \cdots + c_r(E)$*, in* $H_{2(n-r)+\ell}(M; \mathbb{Z})$*, can be realised by a pre-symplectic submanifold* N_k *of height* $n-r$*.*

We provide the uniqueness of constructed submanifolds as follows :

Theorem 3 *The submanifolds constructed as in Theorem 2 are isotopic.*

The next theorem is the analogy of the Lefschetz's hyperplane theorem.

Theorem 4 *Let* N_k *be the pre-symplectic submanifold constructed as in Theorem 2. Then the homotopy morphism* $i_* : \pi_q(N_k) \to \pi_q(M)$ *induced by the inclusion* $i : N_k \to M$ *is the isomorphism for* $q \leq n-r-1$ *and the surjection for* $q = n-r$*. The same statement holds for the homology groups.*

For a $(2n+1)$-dimensional manifold M and a 1-form η on M, a pair (M, η) is a *contact manifold* if $\eta \wedge d\eta^n \neq 0$, and a $(2m+1)$-dimensional submanifold N of M is a *contact submanifold* if $\eta \wedge d\eta^m|_N \neq 0$.

As an application of Theorem 1 and Theorem 2 to contact geometry, we obtain another proof of results by Ibort-Martínez-Presas [4].

Theorem 5 (Ibort-Martínez-Presas [4]) *Let* (M, η) *be a contact* $(2n+\ell)$*-dimensional manifold and* E *a complex vector bundle of rank* r *over* M*,* $(r \leq n)$*. Then the Poincaré dual of* $c_r(E)$*, in* $H_{2(n-r)+1}(M; \mathbb{Z})$*, can be realised by a contact* $(2(n-r)+\ell)$*-dimensional submanifold* N*.*

4. Pre-pencil structures

For a pre-symplectic manifold (M, ω), we say that a coordinate (ϕ, U_x) centered at $x (\in M)$ is a *compatible coordinate* with ω if the restriction $(\phi^*\omega)_0|_{T_0\mathbb{C}^n}$ to $T_0\mathbb{C}^n (\subset T_0(\mathbb{C}^n \times \mathbb{R}^\ell))$ is a positive form of type (1,1), where $\phi : V (\subset \mathbb{C}^n \times \mathbb{R}^\ell) \to U_x (\subset M)$ is a homeomorphism and U_x is a neighborhood of x with $\phi(0) = x$. From now on, we identify U_x with an open set in $\mathbb{C}^n \times \mathbb{R}^\ell$.

We provide a certain pencil structure on a pre-symplectic manifold.

Definition 2 For a pre-symplectic $(2n+\ell)$-dimensional manifold (M, ω) of rank n, a pre-pencil $(A, f, \{\mathcal{P}_\lambda\}_{\lambda \in \Lambda})$ on (M, ω) consists of the following data :

(1) a codimension 4 pre-symplectic submanifold $A (\subset M)$ of height $n-2$,

(2) an ℓ-dimensional submanifold $\{\mathcal{P}_\lambda\}_{\lambda \in \Lambda} (\subset M)$ having a finite number of component (i.e, $|\Lambda| < +\infty$) such that $\mathcal{P}_\lambda$ is the subset of $M \backslash A$ and transverses to any rank $2n$ symplectic subbundle of the tangent bundle TM,

(3) a smooth map $f : M \backslash A \to \mathbb{CP}^1$ whose restriction to $M \backslash (A \cup_{\lambda \in \Lambda} \mathcal{P}_\lambda)$ is a submersion.

Moreover the data have the following standard local models :

(4) at a point $a \in A$ there is a compatible coordinate U_a such that for $(z_1, \ldots, z_n, t_1, \ldots, t_\ell) \in U_a$, A is given by $\{z_1 = z_2 = 0\}$ and f is given by $f(z_1, \ldots, z_n, t_1, \ldots, t_\ell) = z_1/z_2 \in \mathbb{CP}^1$.

(5) at a point of $p \in \mathcal{P}_\lambda$ there is a compatible coordinate U_p and a function γ on $\pi(U_p)$ with $\gamma(0) = 0$ for the projection $\pi : U_p \to \mathbb{R}^\ell$ such that for $(z_1, \ldots, z_n, t_1, \ldots, t_\ell) \in U_p$, f is written as $f(z_1, \ldots, z_n, t_1, \ldots, t_\ell) = f(p) + \gamma(t_1, \ldots, t_\ell) + z_1^2 + \cdots + z_n^2$.

For such a pre-pencil $(A, f, \{\mathcal{P}_\lambda\}_{\lambda\in\Lambda})$, we call the closure of inverse image of a point by f the it fibre of f.

Theorem 6 *Let (M,ω) be a pre-symplectic $(2n+\ell)$-dimensional manifold of rank n. If the class $[\frac{\omega}{2\pi}]$ is integral, then for a sufficiently large integer k there is a pre-pencil $(A, f, \{\mathcal{P}_\lambda\}_{\lambda\in\Lambda})$ on (M,ω) whose fibres are pre-symplectic manifolds of height $n-1$ and realise the Poincaré dual of $[\frac{k\omega}{2\pi}]$ in $H_{2(n-1)+\ell}(M;\mathbb{Z})$.*

References

[1] D.Auroux, *Asymptotically holomorphic families of symplectic submanifolds*, Geom. Funct. Anal. **7** (1997) 971-995.

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[3] S.K.Donaldson. *Lefschetz pencils on symplectic manifolds*, J. Differential Geom. **53** (1999) 205-236.

[4] A.Ibort, D.Martínez, F.Presas. *On the construction of contact submanifolds with prescribed topology*, J. Differential Geom. **56** (2000) 235-283.

Nov. 17th (THU), 11:00–12:00

•**L. Charles** (Paris 6 University)

"On the semi-classical naturality of quantization with half-form"

Geometric quantization is a procedure which associate to any symplectic manifold endowed with a prequantization bundle and a compatible integrable complex structure a quantum space. It is expected that this quantum space doesn't depend on the complex structure, in the sense that there should exist a natural identification between any two quantizations associated to different complex structures.

Ginzburg and Montgomery proved that in many cases such an identification doesn't exist because it would contradict the no go theorems. However since Toeplitz quantization gives a semi-classical representation of the Poisson algebra, this argument doesn't prevent the existence of an identification in the semi-classical limit. Recent results of Foth and Uribe show that even in this case the problem seems to be quite challenging.

I will introduce a slight modification of the quantization including half-form and explain how we can define such a semi-classical identification by using Fourier integral operator. Furthermore I will relate this to parallel transport in the quantum spaces bundle over the space of complex structures endowed with a suitable connection.

Nov. 17th (THU), 13:30–14:30

•**R. Kobayashi** (Nagoya University)

"Toward Nevanlinna/Galois theory of the Gauss map of pseudo-algebraic minimal surfaces"

A complete minimal surface in $\mathbb{R}^3$ is called pseudo algebraic, if its Weierstrass data is defined on a punctured Riemann surface and extends meromorphically to its conformal compactification. In this lecture, I consider the Gauss map lifted to the universal covering surface and study its value distribution from the view point of the Nevanlinna theory coupled with the Galois group action.

Nov. 17th (THU), 15:00–16:00

•**R. Bielawski** (Edinburgh University)

"Asymptotic monopole metrics"

The natural metric on the moduli space of magnetic monopoles of a fixed charge is an example of a complete hyperkaehler metric. These metrics are important for the study of dynamics of magnetic monopoles and for verifying certain duality conjectures in the string theory (Sen's conjecture). In order to understand the L^2-cohomology for these metrics, one needs to understand their asymptotic behaviour when monopoles separate into clusters of monopoles of lower charges. In this talk I will explain how to obtain the asymptotic monopole metrics, corresponding to a cluster decomposition of a monopole, from the flows on compactified Jacobians of singular spectral curves. These metrics should be viewed as a deformation of the product of the monopole metrics of lower charges which captures the interaction of the clusters. I will sketch the proof that the rate of approximation of these metrics is exponential in the separation distance of the clusters, and discuss some applications.

Nov. 18th (FRI), 9:30–10:30

•**Akito Futaki** (Tokyo Institute of Technology)

"Harmonic total Chern forms and stability"

In this talk I will perturb the scalar curvature of Kähler manifolds by incorporarting it with higher Chern forms, and then show that the perturbed scalar curvature has the common propreties as the unpertubed scalar curvature. In particular the perturbed scalar curvature becomes a moment map on the space of all complex structures with a pertubed symplectic structure on a fixed symplectic manifold, which exetnds the results of Fujiki and Donaldson on the unperturbed case.

Nov. 18th (FRI), 11:00–12:00

•**C.-L. Wang** (National Central University)

"Green functions on tori and the mean field equations"

The two dimensional non-linear mean field equation is closely related to the prescribed curvature problem as well as the self-dual condensation of the Chern-Simons-Higgs model. However, even on a flat torus, its solvability depends on the geometry in a non-trivial manner. I this talk I shall report on a recent joint work with C.-S. Lin where we show that the number of critical points of the Green function is exactly the geometric invariant to detect the existence and uniqueness of the mean field equation. We also discuss the geometry of the critical points when the tori vary in the moduli space.

Nov. 18th (FRI), 13:30–14:30

•**J. Rawnsley** (Warwick University)

"Natural star products"

We describe a class of differential star products on symplectic manifolds which we call natural and which includes most constructions of star products. These star products have an associated symplectic connection and a series of closed 2-forms giving their Deligne class as well as allowing the construction of a Fedosov star product to which they are equivalent.

Nov. 18th (FRI), 15:00–16:00

•**H. Upmeier** (University of Marburg)

"Quantization of symmetric spaces, including super-symmetry"

Hermitian symmetric spaces of non-compact type (Cartan domains) play an important role in harmonic analysis (representations of semi-simple Lie groups) and mathematical physics (configuration spaces). Using the Jordan-theoretic description of bounded symmetric domains, we develop a general theory of covariant quantization on Hilbert spaces of holomorphic functions (weighted Bergman spaces) which leads to the so-called geodesic calculus generalizing the well-known Berezin quantization and the Weyl calculus in a uniform way. For the geodesic calculus, we analyze the spectral behaviour of the associated Berezin transform in terms of multivariable hypergeometric functions, and present new results concerning the Moyal products (deformation quantization) in this setting.

The second part of the talk addresses the question of super-symmetric quantization methods, for example for the super unit disk or the super unit ball in any dimension. It is shown that the basic results for the geodesic calculus carry over to the setting of super-holomorphic functions and super-Moebius transformations.

This work is done in collaboration with Professor J. Arazy, University of Haifa, Israel.

Nov. 19th (SAT), 9:30–10:30

•**M. Bordemann** (University of Haute Alsace)

"Quantization of coisotropic submanifolds"

In deformation quantization the structure of the deformed associative function algebra over a Poisson manifold and its isomorphisms is now well-understood thanks to the work by Kontsevitch. An interesting problem, related to the problem of the quantization of (first class) constraints in quantum physics, is the study of certain left or bi-modules of the deformed algebras. In case this module is a function space over a smooth manifold, then this manifold admits a coistropic map into the initial Poisson manifold. We show that even in the symplectic case there are possible obstructions related to the Atiyah-Molino class of the canonical foliation of a coisotropic submanifold of a symplectic manifold. In case this class vanishes (e.g. if the reduced phase space is a smooth manifold) this construction is always possible.

Nov. 19th (SAT), 11:00–12:00

•**T. Mabuchi** (Osaka University)

"Stability on polarized algebraic manifolds"

In this talk, some of our recent results on stability for polarized algebraic manifolds will be given. In particular, we clarify the relationship among K-stability, Hilbert-Mumford stability and Chow-Mumford stability.

Nov. 19th (SAT), 13:30–14:30

•**A. Sergeev** (Steklov Mathematical Institute, Moscow)

"Seiberg-Witten equations on the non-commutative Euclidiean 4-space"

by Alexander POPOV (Joint Institute for Nuclear Research, Dubna),
Armen SERGEEV (Steklov Mathematical Institute, Moscow) and
Martin WOLF (Institut für Theoretische Physik, Hannover)

It is well known that the Seiberg-Witten equations (SW-equations, for short) on a Kähler surface X may have no solutions with a finite action (this is true, e.g., for $X = \mathbb{C}^2 = \mathbb{R}^4$). But, if we introduce a scale parameter λ into these equations then such solutions will appear for sufficiently large $\lambda \geq \lambda_0$. Moreover, these SW_λ-solutions will be parametrized by holomorphic divisors in X.

We consider a non-commutative version of SW-equations on the non-commutative Euclidean 4-space $\mathbb{R}^4_\theta$ and construct non-trivial solutions of these equations which have no commutative limit for $\theta \to 0$ (since SW-equations on $\mathbb{R}^4$ have no non-trivial solutions). So we see that the introduction of the non-commutative factor θ into the SW-equations leads to the same effect as their scale perturbation. It's interesting to compare our SW_θ-solutions with the non-commutative instantons on $\mathbb{R}^4_\theta$, constructed by N.Nekrasov and A.Schwarz. Their instantons also have no commutative limit for $\theta \to 0$ and may be interpreted as non-commutative analogues of singular instantons on $\mathbb{R}^4$ with the curvature, equal to the δ-function, centered at some point of $\mathbb{R}^4$. It seems that, in a similar way, our SW_θ-solutions may be considered as non-commutative analogues of singular SW-solutions on $\mathbb{R}^4 = \mathbb{C}^2$, having the curvature, equal to a current (surface δ-function), concentrated along an algebraic curve in $\mathbb{C}^2$.