Professor Chin-Lung Wang, National Central University, Taiwan, responds to the New Yorker article

November 3, 2006

Dear Mr. Cooper,

The New Yorker article "Manifold Destiny" by Nasar and Gruber on August 28th is an imbalance report with the obvious intention to slander Professor Shing-Tung Yau. I would like to point out some issues raised there with which I'm familiar and where I think the report is completely misleading. This letter grows out as an expanded version (namely with more mathematical details) of the one I sent to Nasar on August 23rd after I saw the preliminary version of their article. Clearly my letter had no influence on their final report. I am among one of the many people who wrote to them before August 28th. When there were still chances for Nasar to listen to different voices she still chose to listen to one side of words. This had disappointed all of us and made us suspect the real purpose of that report. Please freely use this letter in situations where it may be of help.

> 1. *On page 52, "At Harvard, he ran a notoriously tough seminar... Each student was assigned a recently published proof and asked to reconstruct it, fixing any errors and filling in gaps. Yau believed that a mathematician has an obligation to be explicit, and impressed on his students the importance of step-by-step rigor".*

As a former student of Yau at Harvard (1993 – 1998), I benefited a lot from these intense seminars. A significant part of the seminars is to go through some established important theories which were unavailable in courses during that time. Most of us learned from others through it. In fact, we are grateful to Yau for spending so much time with us and sharing with us his insight generously. While some people may consider our seminar tough, many more people in the mathematical community have a great regard for it. Another significant portion of our seminars is to study recent major progresses in Mathematics. This is no doubt the best way to do research. And only in best schools can such activities be run successfully.

By the way, step-by-step rigor is always the necessary ingredient for a mathematical theory to be finally accepted. You may base your research only on results that can be reconstructed. In natural sciences, experimental results can be accepted only if they can be repeated. In Mathematics, a theorem can be accepted only if the proof can be reconstructed without gaps. This is a common sense since the ancient days of Euclid. The above quotation was written purposely to give readers the wrong impression that Yau did not encourage his students to create their own ideas.

> 2. *On p.52 to 53 about the mirror conjecture: "On at least one occasion, Yau and his students have seemed to confuse the two, making claims of originality that other mathematicians believe are unwarranted".*

In the 1980's, the urge to understand the structure of Calabi-Yau manifolds came from two completely different aspects, one in the classification theory in algebraic geometry, another one in string theory. Since then Yau's school at Harvard including his collaborators, post-doctors and students had started an extensive study on this subject. String theory predicts the existence of mirror family of Calabi-Yau manifolds which should give rise to the same quantum field theory as the original Calabi-Yau. In 1991, Candelas et al had made the prediction precise in the case of quintic three-folds which states that the pre-potential of instanton invariants should agree with the pre-potential of special geometry on the complex structure moduli space of the mirror quintics.

The progress in proving the mirror prediction and in particular the formula of Candelas et al is best explained by the existing literatures following the time order, and from which one finds promptly that the above quotation is baseless:

Givental's article in 1996 was the first claiming to prove the mirror prediction. Givental's argument involves Floer homology and the theory of equivariant quantum cohomology, experts did find difficulties in following his argument. Manin in his 1996 Max Plank preprint said that "some work remains to be done in order to complete his arguments".

In 1997, Lian, Liu and Yau published their proof of the mirror prediction and introduced a number of new ideas. The first one is the concept of Euler data which enabled them to give a direct argument and made their results applicable to many important situations beyond the formula of Candelas et al. Another crucial new idea in LLY is the special geometry relation between the one-point invariants and the instanton pre-potential. The Mathematical Review MR1621573 (99e:14062) of LLY by Gathmann said that "however, his (Givental's) proof was hard to understand and at some points incomplete. The current paper of Lian, Liu, and Yau now gives the first complete rigorous proof of the physicists' formula".

After the appearance of LLY, Pandharipande's 1998 report in Seminar Bourbaki (math.AG/9806133) explains some of Givental's argument. It says that "A complete proof of the Mirror prediction for quintics by Lian, Liu, and Yau using localization formulas has appeared recently in [LLY]. The argument announced by Givental in [G1] yields a complete proof of (i)-(iii)". The AMS book by Cox and Katz in 1999 explains some details of both proofs. This book follows LLY's proof closely, but in discussing Givental's proof details are often sketchy or referred to Pandharipande's report instead.

The work of LLY is of course not totally independent of earlier works. It was written explicitly in their introduction that their Mirror Principle is a combination of ideas initiated and developed by many people including Kontsevich, Givental, Witten and Candelas et al. Indeed Givental's work was also inspired by Kontsevich and others. No one works totally independently and we all rely on one another. Most mathematicians give credit to both papers since their viewpoints are all very helpful to later developments in this field.

I would like to highlight some importance feature of Euler data through one example. Already in the LLY paper Euler data was applied to concave bundles and to the study of local mirror symmetry. The concave bundle case was also handled by Givental in his subsequent papers dated after LLY. In my earlier investigation on the invariance of small quantum ring under simple flops in 2004, Euler data for three-point functions for concave bundles plays a decisive role in forming the philosophical basis that such an invariance statement is possible. The final proof (math.AG/0608370, joint with Lee and Lin) turns out can be carried out inductively. The starting point then can be based on either LLY's result on one-point invariants or Givental's. Yet without the earlier inspiration of Euler data, my research on this problem will probably have not started.

3. *On p.52, "More than a decade had passed since Yau had proved his last major result".*

I am not sure how such a ridiculous statement is drawn (by a reporter?!). It is already funny to identify "his last major result". Maybe they meant the existence of complete Kaehler-Einstein metrics in late 1980's, or maybe they meant the Donaldson-Uhlenbeck-Yau theorem on the existence of Hermitian-Yang-Mills connections on stable bundles. This later result, proved by Donaldson in 1985 for the two dimensional case and Uhlenbeck-Yau in 1986 for general dimensions is indeed of fundamental importance in algebraic geometry as well as in physics. The theorem became much more prominent in the 1990's. In physics literatures, they call it DUY theorem. Recently in 2004 Li and Yau solved an important system of equations of Strominger based on the HYM-DUY connections.

On the other hand, it is well-known that in 1996 Strominger, Yau and Zaslow initiated the so-called SYZ program which provides a ground-breaking insight into the structure of Calabi-Yau manifolds. Together with Hamilton's Ricci flow, which Yau has played a significant role in its development without asking for a formal credit (see Hamilton's letter for Yau's insight on Ricci flow), they form the most important two subjects in differential geometry in the past ten years. While Yau does prove many major results (including the mirror prediction) in recent years, as a world leading figure in geometry he also directed and promoted frontier researches more than anyone else in this field.

Before SYZ, researches on general Calabi-Yau manifolds are limited to algebro-geometric methods and there was indeed no satisfactory mathematical explanation of the full mirror symmetry phenomenon. It was also unsatisfactory that mathematicians play only the role to verify (though highly non-trivially) the physics predictions or even just to make their predictions mathematically meaningful. Geometers seemed to lack of some fundamental thinking in attacking these problems. It was the SYZ program which took the major task to reveal the underlying structure of Calabi-Yau manifolds and brought together other fields in mathematics including differential geometry and non-linear analysis into the study. This approach opens new research directions in geometry and has since then been very fruitful.

Calabi-Yau manifolds serve as the underlying spaces of string theory, the expected theory of everything. Yau's solution to the Calabi conjecture in the 1970's made possible the starting of this story. Twenty years later, the SYZ program made a truly mathematical input to this story again. Mathematics and theoretic physics have influenced each other deeply in this exciting historical moment. Without the tremendous efforts made by Yau and his collaborators the story may not have been as exciting as it is now.

Sincerely yours,

Chin-Lung Wang
Professor of Mathematics
National Central University
Chung-Li 32051, Taiwan
dragon@math.ncu.edu.tw