①

Cohomological field theory (CohFT)

Let $q: \overline{\mathcal{M}}_{g-1,n+2} \to \overline{\mathcal{M}}_{g,n}$ be the map sewing p_{n+1} and p_{n+2}

$s: \overline{\mathcal{M}}_{g_1,n_1+1} \times \overline{\mathcal{M}}_{g_2,n_2+1} \to \overline{\mathcal{M}}_{g,n}$ ($= g_1+g_2,\ n_1+n_2$) be the map sewing p_{n_1+1} on the 1st comp. and p_{n_2+1} on the 2nd comp.

$p: \overline{\mathcal{M}}_{g,n+1} \to \overline{\mathcal{M}}_{g,n}$ forgetting p_{n+1}

Let $A \in \mathrm{FinVect}_{\mathbb{C}}$, $\mathbb{1} \in A$ nonzero

$\eta = \eta_{\mu\nu} e^{\mu} \otimes e^{\nu}$ sym. bilinear form on A, nondegenerate

A <u>nodal</u> CohFT of $(A, \eta, \mathbb{1})$ is a set of linear maps

$\overline{\Omega}_{g,n}: A^{\otimes n} \to H^{\bullet}(\overline{\mathcal{M}}_{g,n})$, $2g-2+n>0$ s.t.

1. $\overline{\Omega}_{g,n}$ is S_n-equivariant $\forall g, n$
2. $\overline{\Omega}_{0,3}(\mathbb{1} \otimes u \otimes v) = \eta(u,v) \quad \forall u, v \in A$
3. $q^{*}\overline{\Omega}_{g,n}(v_1 \otimes \cdots \otimes v_n) = \eta^{\mu\nu}\overline{\Omega}_{g-1,n+2}(v_1 \otimes \cdots \otimes v_n \otimes e_\mu \otimes e_\nu)$
4. $s^{*}\overline{\Omega}_{g,n}(v_1 \otimes \cdots \otimes v_n)$
$= \eta^{\mu\nu}\overline{\Omega}_{g_1,n_1+1}(v_1 \otimes \cdots \otimes v_{n_1} \otimes e_\mu) \times \overline{\Omega}_{g_2,n_2+1}(v_{n_1+1} \otimes \cdots \otimes v_n \otimes e_\nu)$
5. $p^{*}\overline{\Omega}_{g,n}(v_1 \otimes \cdots \otimes v_n) = \overline{\Omega}_{g,n+1}(v_1 \otimes \cdots \otimes v_n \otimes \mathbb{1})$

$\left(= \widetilde{\mathcal{M}}_{g,n} \text{ when } I = [n] \right)$

Let $\pi_I: \widetilde{\mathcal{M}}^{I}_{g,n} \to \overline{\mathcal{M}}_{g,n}$, $I \subset [n]$ be the torus bundle with fiber at $C \in \overline{\mathcal{M}}_{g,n}$ be $\prod_{i \in I} S(T_{p_i} C)$

$S(V) = (V \setminus \{0\}) / \mathbb{R}^{+}$

Denote $p: \widetilde{\mathcal{M}}_{g,n+1} \to \widetilde{\mathcal{M}}_{g,n}$ forgetting p_{n+1}