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Ex 9.0.1.

$$Z(\alpha, \epsilon) = \int dX e^{-\frac{\alpha}{2}X^2 - i\epsilon X^3} = \int dX \sum_{n=0}^{\infty} e^{-\frac{\alpha}{2}x^2} \frac{(-i\epsilon X^3)^n}{n!} = \sum_{n=0}^{\infty} (-i\epsilon)^n \int dx\, e^{-\frac{\alpha}{2}x^{2}} X^{3n}/n!$$

To show $Z(\alpha, \epsilon) = \exp\left(\sum_{G \text{ conn.}} (-3!\, i\epsilon)^{|V|} \alpha^{-|E|}/|Aut(G)|\right)$

$$= \exp\left(\sum_{\substack{G \text{ conn.} \\ \text{labelled}}} (-i\epsilon)^{|V|} \alpha^{-|E|}/|V|!\right)$$

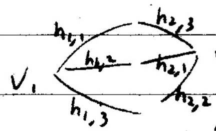
(Sum over graphs with labelled vertices ~~and their hands~~)

(by expanding the exp)

$$= \sum_{\substack{G_1, \ldots, G_r \\ \text{Conn. labelled}}} (-i\epsilon)^{|V|} \alpha^{-|E|} \frac{1}{r!\,|V_1|! \cdots |V_r|!}$$

$$(V = \coprod V_i,\ E = \coprod E_i)$$

$$= \sum_{\substack{G \text{ labelled} \\ (\text{conn. or not})}} (-i\epsilon)^{|V|} \alpha^{-|E|}/|V|!$$



An example of labelled graph

Each unlabelled graph is counted by $(3!)^{|V|} \frac{|V|!}{|Aut(G)|}$ labelled graphs

Each general labelled graph with labelled conn. comp. counted by $\frac{|V|!}{|V_1|! \cdots |V_r|!}$ r-tuples of conn. labelled graphs

this gives the factor $r!$

$$= \sum_{n=0}^{\infty} (-i\epsilon)^n \alpha^{-\frac{3n}{2}} \times \underbrace{\sum_{\substack{G \text{ labelled} \\ |V| = n}} 1}_{= \# \text{ of ways of contracting } n \text{ 3-stars}}$$

Now this follows from the intepretation of ways of contracting

(Note that when $\underline{n \text{ is odd}}$ there is no way to contract by handshaking lemma)

Suppose the CVF is proved for $n-1$ variables.
Then for n variables, we first prove that CVF is true when changing one variable. Let $x_1, x_2, \cdots, \varphi_n$ be the var.
If x_1 is replaced by $\tilde{x}_1 = h_1 + h_2\varphi_n$, h_1, h_2 involve no φ_n, then we want to prove

$$(1)\quad \left(\int f(x_1,\cdots)\,dx_1\cdots d\varphi_n = \right)\int f(\tilde{x}_1,\cdots)\,d\tilde{x}_1\cdots d\varphi_n$$
$$\overset{?}{=} \int f(h_1+h_2\varphi_n)\,\mathrm{Ber}\,J\,dx_1\cdots d\varphi_n \quad \text{(we omit other var. in } f\text{)}$$

Here $\mathrm{Ber}\,J = \frac{\partial h_1}{\partial x_1} + \frac{\partial h_2}{\partial x_1}\varphi_n$. By expanding f we have

$$\int f(h_1+h_2\varphi_n)\left(\frac{\partial h_1}{\partial x_1}+\frac{\partial h_2}{\partial x_1}\varphi_n\right)dx_1\cdots d\varphi_n$$
$$= \int \big(f(h_1) + h_2\varphi_n f'(h_1)\big)\big(h_1' + h_2'\varphi_n\big)\,dx_1\cdots d\varphi_n$$

(we write prime ($'$) for $\frac{\partial}{\partial x_1}$)

$$= \int f(h_1)h_1' + f(h_1)h_2'\varphi_n + h_2\varphi_n f'(h_1)h_1'\,dx_1\cdots d\varphi_n$$

By hypothesis we have $\int f(h_1)h_1'\,dx_1\cdots d\varphi_n$
$= \int f(h_1)\,dh_1\cdots d\varphi_n = \int f(x_1,\cdots)\,dx_1\cdots d\varphi_n$

And IBP shows that

$$\int f(h_1)h_2'\,dx_1\cdots d\varphi_n = \int f(h_1)\,dh_2\cdots d\varphi_n$$
$$= -\int h_2\,d(f(h_1))\cdots d\varphi_n = -\int h_2 f'(h_1)h_1'\,dx_1\cdots d\varphi_n$$

Thus $\int f(h_1)h_2'\varphi_n\,dx_1\cdots d\varphi_n = \int h_2 f'(h_1)h_1'\varphi_n\,dx_1\cdots d\varphi_n$ by comparing the φ_n part. Thus (1) is true.

On the other hand if φ_n is replaced by $\tilde{\varphi}_n = h_1 + h_2\varphi_n$, h_i involves no φ_n, then we want to prove

$$(2)\quad \left(\int f(\cdots,\varphi_n)\,dx_1\cdots d\varphi_n =\right)\int f(\cdots,\tilde{\varphi}_n)\,dx_1\cdots d\tilde{\varphi}_n$$
$$\overset{?}{=} \int f(h_1+h_2\varphi_n)\,\mathrm{Ber}\,J\,dx_1\cdots d\varphi_n$$

Here $\mathrm{Ber}\,J = h_2^{-1}$. Write $f = g_1 + g_2\varphi_n$, then (g_i involves no φ_n)

$$\int f(h_1+h_2\varphi_n)h_2^{-1}\,dx_1\cdots d\varphi_n = \int g_1 + g_2(h_1+h_2\varphi_n)h_2^{-1}\,dx_1\cdots d\varphi_n$$
$$= \int g_2\varphi_n\,dx_1\cdots d\varphi_n = \int g_1+g_2\varphi_n\,dx_1\cdots d\varphi_n = \int f\,dx_1\cdots d\varphi_n$$

Hence (2) is also true. Now since Ber is multiplicative, we can prove CVF by changing variables ~~one~~ by one.

Work out the P.O.U. argument for the general case.

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Ex 2.2.

Note that $[x,p]=i$

$\Rightarrow x^n p = p x^n + i n x^{n-1}$ by induction

$\Rightarrow [f(x), p] = i f'(x)$

$[f(x), p^2] = i f'(x) p + p i f'(x)$

$\boxed{\begin{array}{l}(\bar\psi\psi - \psi\bar\psi)\bar\psi = \bar\psi \\ \bar\psi(\bar\psi\psi - \psi\bar\psi) = -\bar\psi\end{array}}$

Thus $[H, Q] = [\frac{1}{2}p^2 + \frac{1}{2}(h'(x))^2 + \frac{1}{2}h''(x)(\bar\psi\psi - \psi\bar\psi)), \bar\psi(ip + h'(x))]$

$= \bar\psi([\frac{1}{2}p^2, h'(x)] + [\frac{1}{2}(h'(x))^2, ip])$

$+ \frac{1}{2}\bar\psi(h''(x) ip + ip h''(x)) + \bar\psi h''(x) h'(x) = 0$

Similarly $[H, \bar Q] = -[H, Q]^\dagger = 0$ since H is real $(H^\dagger = H)$

We also check $\delta\mathcal{O} = [\epsilon Q + \bar\epsilon \bar Q, \mathcal{O}]$

$[\epsilon Q + \bar\epsilon\bar Q, x] = [\epsilon\bar\psi(ip + h'(x)), x] + [\bar\epsilon\psi(-ip + h'(x)), x]$

$= \epsilon\bar\psi - \bar\epsilon\psi = \delta x$

$[\epsilon Q + \bar\epsilon\bar Q, \psi] = [\epsilon\bar\psi(ip + h'(x)), \psi] + [\bar\epsilon\psi(-ip + h'(x)), \psi]$

$= \epsilon(ip + h'(x))$ (Note that ϵ $(\bar\epsilon)$ and ψ are anticom.

$= \delta\psi$ and $\psi\bar\psi + \bar\psi\psi = 1$)

$[\epsilon Q + \bar\epsilon\bar Q, \bar\psi] = [\epsilon Q + \bar\epsilon\bar Q, \psi]^\dagger = \bar\epsilon(-ip + h'(x)) = \delta\bar\psi$ com.

since $\epsilon Q + \bar\epsilon\bar Q$ is hermitian $(\hat\delta^\dagger = -\hat\delta)$

$((\epsilon Q + \bar\epsilon\bar Q)^\dagger = \bar Q\bar\epsilon + Q\epsilon = -\epsilon Q - \bar\epsilon\bar Q)$

Product Rule : $\delta O_1 O_2 = (\delta O_1) O_2 + O_1 \delta O_2$

$= [\hat\delta, O_1] O_2 + O_1 [\hat\delta, O_2]$

$= [\hat\delta, O_1 O_2]$

Thus $\delta\mathcal{O} = [\hat\delta = \epsilon Q + \bar\epsilon\bar Q, \mathcal{O}]$ holds for all $\mathcal{O} = \mathcal{O}(x, \psi, \bar\psi)$

Let $H = \hat{p}^2/2m + V(\hat{q})$ and $L(\dot{q}, q) = \frac{1}{2}m\dot{q}^2 - V(q)$

We want to show this path integral representation

$\langle q_F | e^{-iHT} | q_I \rangle = \int Dq(t)\, e^{i\int_0^T dt\, L(\dot{q}, q)}$ q_I : initial position

Write $\langle q_F | e^{-iHT} | q_I \rangle$ q_F : final position

$$= \left(\prod_{j=1}^{N-1} \int dq_j\right) \langle q_F | e^{-iH\frac{T}{N}} | q_{N-1} \rangle \cdots \langle q_1 | e^{-iH\frac{T}{N}} | q_0 \rangle$$

where we use the resolution of identity:

$$\int dq_j\, |q_j\rangle\langle q_j| = I$$

For each $\langle q_j | e^{-iH\frac{T}{N}} | q_{j-1} \rangle$ we write

$$\langle q_j | e^{-iH\frac{T}{N}} | q_{j-1} \rangle = \int \frac{dp}{2\pi} \langle q_j | e^{-iH\frac{T}{N}} | p \rangle \langle p | q_{j-1} \rangle$$

where $|p\rangle$ are eigenstates of $\hat{p}$ and $\langle p|$ are normalized so that $\int \frac{dp}{2\pi} |p\rangle\langle p| = I$

Now $\langle q_j | e^{-iH\frac{T}{N}} | p \rangle$ " = " $\left(\langle q_j | e^{-iV(\hat{q})}\right)\left(e^{-i\hat{p}^2/2m} | p \rangle\right) + O(\frac{1}{N^2})$

$$= e^{-ip^2/2m} e^{-iV(q_j)} \langle q_j | p \rangle + O(\tfrac{1}{N^2}) \quad \text{as } N \to \infty$$

(Using Zassenhaus formula:

$$e^{\frac{T}{N}(X+Y)} = e^{\frac{T}{N}X} e^{\frac{T}{N}Y} e^{\frac{T^2}{2N^2}[X,Y]} \cdots \quad \text{for any operator } X, Y$$

$1 + O(\frac{1}{N^2})$)

Hence the integral gives (use the relation $\langle p | q \rangle = e^{ipq}$)

$$\langle q_j | e^{-iH\frac{T}{N}} | q_{j-1} \rangle = e^{-iV(q_j)} \int e^{-ip^2/2m + ip(q_j - q_{j-1})} \frac{dp}{2\pi} + O(\tfrac{1}{N^2})$$

$$= e^{-iV(q_j)} \left(\frac{-im}{2\pi \frac{T}{N}}\right)^{\frac{1}{2}} e^{im(q_j - q_{j-1})^2/2\frac{T}{N}} + O(\tfrac{1}{N^2})$$

$\prod_j$ gives $\langle q_F | e^{-iHT} | q_I \rangle = \underbrace{\left(\frac{-im}{2\pi \frac{T}{N}}\right)^{\frac{N}{2}} \left(\prod_{j=1}^{N} dq_j\right)}_{\text{by def}} e^{\frac{T}{N}\sum_{j=1}^{N}\left(-iV(q_j)/\frac{T}{N} + im(q_j - q_{j-1})^2/2\frac{T^2}{N^2}\right)} + O(\frac{1}{N})$

$$\xrightarrow{N\to\infty} \;=\; \int Dq(t)\, e^{\int_0^T -iV(q) + \frac{im\dot{q}^2}{2}\, dt}$$

$$= \int Dq(t)\, e^{i\int_0^T L(\dot{q}, q)\, dt}$$

$\int$ via P.V.I

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Ex 12.1.3.

1. $\bar{D}_+\bar{D}_-\mathcal{F}=0 \Rightarrow \mathcal{F}=G_+ + G_-$ with $\bar{D}_+G_+=0$, $\bar{D}_-G_-=0$:

By Poincare Lemma,

$\bar{D}_+\bar{D}_-\mathcal{F}=0 \Rightarrow \bar{D}_-\mathcal{F}=\bar{D}_+G$ for some superfield G

$\Rightarrow D_-\bar{D}_-\mathcal{F}=D_-\bar{D}_+G$

$\Rightarrow 2i\partial_-F-\bar{D}_-D_-\mathcal{F}=D_-\bar{D}_+G$

$\Rightarrow F=\frac{1}{2i}\int_{-\infty}^{x^-}(\bar{D}_-D_-\mathcal{F}+\bar{D}_+D_-G)\,dx'^-$

thus we can take $G_+=\frac{1}{2i}\int_{-\infty}^{x^-}D_-G\,dx'^-$

$G_-=\frac{1}{2i}\int_{-\infty}^{x^-}D_-F\,dx'^-$

The case D_+D_-, $\bar{D}_+D_-$, $D_+\bar{D}_-$ are similar

2. $\bar{D}_+\Phi=\bar{D}_-\Phi=0 \Rightarrow \Phi=\bar{D}_+\bar{D}_-\mathcal{E}$ for some superfield $\mathcal{E}$:

By Poincare Lemma,

$\Phi=\bar{D}_+G$ for some G and $\bar{D}_+\bar{D}_-G=-\bar{D}_-\bar{D}_+G=0$

By 1., $G=G_++G_-$ with $\bar{D}_+G_+=0$, $\bar{D}_-G_-=0$

$\Rightarrow \Phi=\bar{D}_+G_-=\bar{D}_+\bar{D}_-\mathcal{E}$ for some superfield $\mathcal{E}$

For twisted chiral field it is similar. (again by Poincare Lemma)

3. $\bar{D}_+\bar{D}_-\mathcal{F}=D_+D_-\mathcal{F}=0$

$\Rightarrow \mathcal{F}=U_1+\bar{U}_2$ with $U_1=\bar{D}_+D_-H_1$, $\bar{U}_2=D_+\bar{D}_-H_2$: (twisted chiral)

By 1., $\mathcal{F}=\bar{D}_+H_++\bar{D}_-H_-$ for some H_+, H_-

and poincare lemma,

$D_+D_-\bar{D}_+H_++D_+D_-\bar{D}_-H_-=0$

$\Rightarrow D_-\bar{D}_+H_++D_-\bar{D}_-H_-=D_+G$ for some G

$\Rightarrow H_-=\frac{1}{2i}\int_{-\infty}^{x^-}(-D_-\bar{D}_+H_++\bar{D}_-D_-H_-+D_+G)\,dx'^-$

$\Rightarrow \mathcal{F}=\bar{D}_+\left(H_++\frac{1}{2i}\int_{-\infty}^{x^-}\bar{D}_-D_-H_+\,dx'_-\right)$

$+\bar{D}_-\left(H_-+\frac{1}{2i}\int_{-\infty}^{x^-}D_-\bar{D}_+H_+\,dx'_-\right)$

$=\bar{D}_+\left(\frac{1}{2i}\int_{-\infty}^{x^-}D_-\bar{D}_-H_+\,dx'\right)$

$+\bar{D}_-\left(\frac{1}{2i}\int_{-\infty}^{x^-}\cancel{D_+D_-H_+}+D_+G\right)dx'^-$

thus we can take $H_1=\frac{1}{2i}\int_{-\infty}^{x^-}\bar{D}_-H_+\,dx'_-$

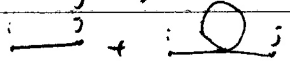
$H_2=\frac{1}{2i}\int_{-\infty}^{x^-}D_+G\,dx'_-$

GEE-JUMP

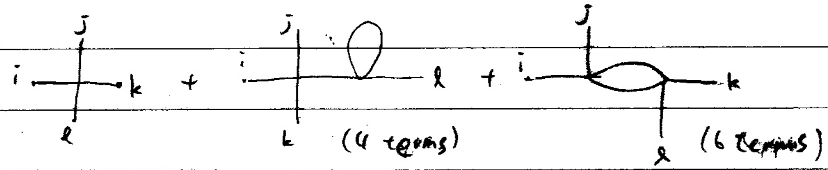
Ex 14.2.1.

By Feynman Diagram,

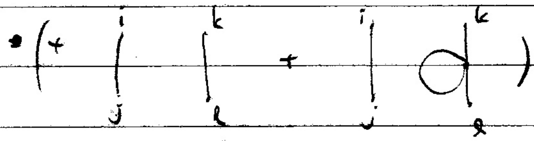
Assume C_{klmn} symm. on indices

$\langle x_i, x_j \rangle_{(1)} =$ 

$= (M^{-1})^{ij} + 12 (M^{-1})^{ik} (M^{-1})^{jl} (M^{-1})^{mn} C_{klmn}$

$\langle x_i, x_j, x_k, x_l \rangle_{(1)} =$ 

(4 terms) (6 terms)



(disc.)

(3 terms) (6 terms)

0-loop { disc. $= (M^{-1})^{ij} (M^{-1})^{kl} + (M^{-1})^{ik} (M^{-1})^{jl} + (M^{-1})^{il} (M^{-1})^{jk}$

conn. $+ 24 (M^{-1})^{ip} (M^{-1})^{jq} (M^{-1})^{kr} (M^{-1})^{ls} C_{pqrs}$

1-loop { disc. $+ 12 (M^{-1})^{ij} (M^{-1})^{kp} (M^{-1})^{lq} (M^{-1})^{rs} C_{pqrs}$ (+ other 5 terms)

conn. $+ 288 (M^{-1})^{ip} (M^{-1})^{jq} (M^{-1})^{kr} (M^{-1})^{st} (M^{-1})^{lu} (M^{-1})^{vw}$

$C_{pqrs} C_{tuvw}$ (+ other three terms)

$+ 288 (M^{-1})^{ip} (M^{-1})^{jq} (M^{-1})^{kt} (M^{-1})^{lu} (M^{-1})^{rv} (M^{-1})^{sw}$

$C_{pqrs} C_{tuvw}$ (+ other five terms)

Lemma (Wick) If we write $\langle f \rangle = \int f e^{-\frac{1}{2} x_i M_{ij} x_j} \prod dx_i \Big/ \int e^{-\frac{1}{2} x_i M_{ij} x_j} \prod dx_i$ $(M>0)$

then for any linear forms $f_1, \cdots f_{2n}$ we have

$$\langle f_1 \cdots f_{2n} \rangle = \sum \langle f_{p_1} f_{q_1} \rangle \cdots \langle f_{p_n} f_{q_n} \rangle \quad (*)$$

where p_i, q_i is a permutation of $(2n)$ s.t.

$p_1 < \cdots < p_n$, $p_i < q_i$ for $i = 1, \cdots, n$ (i.e., all pairings)

Proof. By change of variables may assume $M = I$

then since both side of $(*)$ is linear in each f_i,

we only have to check for monomials $f_i = x_{k_i}$ for some k_i

Since $\langle x_i x_j \rangle = \delta_{ij}$ $\forall i, j$, it suffices to show

$\langle x_i^{2n_i} \rangle = (2n_i - 1)!! \langle x_i^2 \rangle^{n_i} (= (2n_i - 1)!!$, # of pairings of $[2n_i]$)

If so, then $\langle x_1^{n_1} \cdots x_k^{n_k} \rangle = \prod_i \langle x_i^{n_i} \rangle$ by separating variables

and we are done (Note that $\langle x_i^{n_i} \rangle = 0$ if n_i is odd,

and all pairing gives 0 for such product)

But this can be easily shown by integration by part & recurrence

$$\int x^{2n_i} e^{-\frac{1}{2}x^2} dx = (2n_i - 1) \int x^{2n_i - 2} e^{-\frac{1}{2}x^2} dx$$

Now with this lemma, the integral

$$\langle \mathcal{O} \rangle_{(L)} = \int \prod_{i=1}^{n} dx_i \exp(-\tfrac{1}{2} x_i M_{ij} x_j + C_{ijkl} x_i x_j x_k x_l) \mathcal{O}(x_1, \cdots x_n)$$
$$\Big/ \int \prod_{i=1}^{n} dx_i \exp(-\tfrac{1}{2} x_i M_{ij} x_j) \qquad (L = 0, 1)$$

can be calculated with giving each Feymann diagram

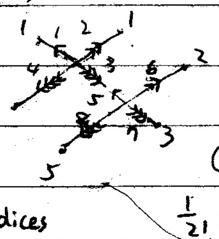
the weight C_{ijkl} for each interaction vertex

$(M^{-1})^{ij} (= \langle x_i x_j \rangle)$ for each propagator

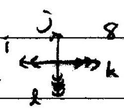
(Note that for any diagram with no endpoint,

and having at least one vertex, it has at least

two loops, and hence the denominator has ~~no~~ C_{ijkl} terms)

Example: 

This graph has value $(M^{-1})^{11} (M^{-1})^{12} (M^{-1})^{48} (M^{-1})^{35} (M^{-1})^{58} (M^{-1})^{34}$
$(M^{-1})^{37} C_{1234} C_{5678}$

(Here I use 

to distinguish the indices
of interaction vertices)

Contributes to $\int \prod_{i=1}^{n} dx_i \exp(\frac{1}{2} x_i M_{ij} x_j$
$\frac{1}{2!} C_{klmn} x_k x_l x_m x_n C_{pqrs} x_p x_q x_r x_s)$

We do the analog computation for

$$\langle \xi^I(x)\,\xi^J(y)\rangle_{(1)} \quad \text{and} \quad \langle \xi^I(x)\,\xi^J(y)\,\xi^K(z)\,\xi^L(w)\rangle_{(1)}$$

for the nonlinear sigma model expanded at a point:

$$S = \tfrac{1}{2}\int \partial^\mu \xi^I \partial_\mu \xi_I - \tfrac{1}{3} R_{IKJL}\,\xi^K \xi^L \partial^\mu \xi^I \partial_\mu \xi^J + O(\xi^5)\, d^2x$$

We find the matrix M_{ij} and C_{ijkl} in momentum space p_i (here the index "i" parametrize both the momentum and the indices of bosonic variables) (i, I)

$$M_{ij} = p_i^2\,\delta^{(2)}(p_i+p_j)\,\delta(I,J), \qquad M^{-1}_{ij} = \frac{1}{p_i^2}\,\delta^{(2)}(p_i+p_j)\,\delta(I,J)$$

$$C_{ijkl} = \tfrac{1}{6}\, p_i\cdot p_j\, R_{IKJL}\,(2\pi)^2\,\delta^{(2)}(p_i+p_j+p_k+p_l)$$

(This is because $\int e^{ip_i q}\cdot e^{ip_j q}\cdot e^{ip_k q}\cdot e^{ip_l q}\, dq$

$$= \int e^{i(p_1+\cdots+p_4)q} = (2\pi)^2\,\delta^{(2)}(p_1+\cdots+p_4)$$

(physical, the net momentum is zero)

Thus the Feynman rules gives

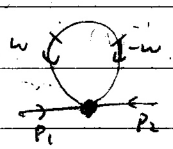
$$\langle \xi^I(x),\, \xi^J(y)\rangle_{(0)} = \iint \frac{d^2p_1}{(2\pi)^2}\, e^{-ip_1 x}\, \frac{d^2p_2}{(2\pi)^2}\, e^{-ip_2 y}\, \delta^{(2)}(p_1+p_2)\,\delta(I,J)\,/\,p_1^2$$

$$= \int \frac{d^2p_1}{(2\pi)^2}\, \frac{e^{-ip_1(x-y)}}{p_1^2}\,\delta(I,J)$$

$$\langle \xi^I(x)\,\xi^J(y)\rangle_{(1)} = \text{—} + \iiiint \frac{d^2p_1}{(2\pi)^2}\, e^{-ip_1 x}\, \frac{d^2p_2}{(2\pi)^2}\, e^{-ip_2 y}\, \frac{1}{p_1^2}\,\frac{1}{p_2^2}$$

$$\cdot\, 2\cdot \int \tfrac{1}{6}\, p_1\cdot p_2\, R_{IKJL}\,(2\pi)^2\,\delta^{(2)}(-p_1-p_2+k+(-k))$$

$$\cdot\, \frac{1}{k^2}\,\delta(K,L)\, d^2k \qquad (p_1 = -p_2)$$



$$= \text{—} + \int \frac{d^2p_1}{(2\pi)^2}\, \frac{e^{-ip_1(x-y)}}{p_1^2}\left(\frac{1}{3}\int \frac{d^2k}{(2\pi)^2}\,\frac{1}{k^2}\, R_{IJ}\right)$$

Note that this is only 2 of 12 possible pairing

8 of them ~~are~~ cancelled by oddity

and the last two terms involve $\int \frac{d^2k}{(2\pi)^2} R_{IJ}$ ..

hence diverges, and the book simply ignore them (footnote of P321)

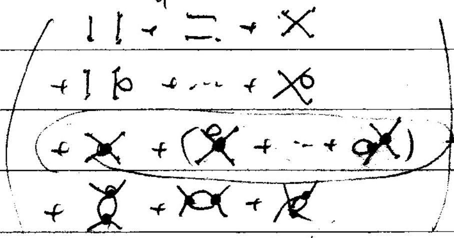
Need to add 2 more terms (tachyon & dilaton)

into the action to eliminate this in renormalization

(see §3.2 in QF & Strings vol 2)

similary for four point we have

$\langle \xi^{I_1}(x_1)\cdots\xi^{I_4}(x_4)\rangle_{(1)}$



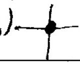
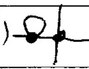
For this part $= \int \prod_{i=1}^{4} \frac{d^2p_i}{(2\pi)^2}\frac{e^{-ip_ix_i}}{p_i^2}$

$\Big(2\cdot p_3\cdot p_4\cdot\frac{1}{6}R_{J I_3 I_2 I_4}\,\delta^{(2)}(p_1+p_2+p_3+p_4)(2\pi)^2$

$\Big(\delta(J,I_1) + \int\frac{d^2k}{(2\pi)^2}\frac{1}{k^2}R^{J}_{I_1\,J}\Big)$ replace one of $I_1 I_3 I_2 I_4$ by J

as 2-point case have shown

and thus these terms indeed occurs.

(I_1,x_1)  and (I_1,x_1) 

has common factor $\frac{1}{p_1^2}$ on the left and can be taken out

15.1.1. Plug $(\phi_i(x)) \to (e^{i\gamma(x)}\phi_i(x))$

into $v_\mu(x) = \frac{i}{2}\sum_{i=1}^N (\bar\phi_i \partial_\mu \phi_i - \partial_\mu \bar\phi_i \phi_i) / \sum|\phi_i|^2$

we obtain $v_\mu(x) \nearrow$ $\frac{i}{2}\sum_{i=1}^N \Big(e^{-i\gamma(x)}\bar\phi_i \partial_\mu (e^{i\gamma(x)}\phi_i(x))$

$- \partial_\mu (e^{-i\gamma(x)}\bar\phi_i) e^{i\gamma(x)}\phi_i\Big) / \sum|\phi_i|^2$

$= \frac{i}{2}\sum_{i=1}^N (\bar\phi_i \partial_\mu \phi_i + i\partial_\mu\gamma(x)\bar\phi_i(x)\phi_i(x)$

$- \partial_\mu\bar\phi_i \phi_i + i\partial_\mu\gamma(x)\bar\phi_i(x)\phi_i(x)) / \sum|\phi_i|^2$

$= v_\mu(x) - \partial_\mu \gamma(x)$

Put $D_\mu \phi_i = (\partial_\mu + i v_\mu)\phi_i$ $(D_\mu\bar\phi_i = \overline{D_\mu\phi_i} = (\partial_\mu - i v_\mu)\bar\phi_i)$

into the equation

$\underline{\sum_1^N (D_\mu\bar\phi_i \phi_i - \bar\phi_i D_\mu \phi_i) = 0}$ where does it come from?

gives $\sum_{i=1}^N (\phi_i \partial_\mu \bar\phi_i - i v_\mu \bar\phi_i \phi_i$

$- \bar\phi_i \partial_\mu \phi_i - i v_\mu \bar\phi_i \phi_i) = 0$

$\Rightarrow v_\mu = \frac{i}{2}\sum_{i=1}^N (\bar\phi_i \partial_\mu \phi_i - \partial_\mu \bar\phi_i \phi_i) / \underset{|\phi_i|^2}{\underset{\shortparallel}{\sum_{i=1}^N \bar\phi_i \phi_i}}$

Ex 19.1.1. $S = \int_\Sigma \partial^\mu \phi \, \partial_\mu \phi \, d^2x$

$\delta S = \int_\Sigma 2 \partial^\mu \phi \, \partial_\mu \delta \phi \, d^2x$

$= \int_\Sigma 2 \partial^\mu (\cdots) - 2\phi \, \partial^\mu \partial_\mu \phi$

$= \int_{\partial \Sigma}$

Ex 19.1.1. $S = \int_\Sigma \partial_\mu \phi \, \partial^\mu \phi \, d^2x$

$$0 = \delta S = \int_\Sigma 2 \partial_\mu \delta\phi \, \partial^\mu \phi \, d^2x$$

$$= \int_\Sigma \left(2\partial_\mu (\delta\phi \, \partial^\mu \phi) - 2\delta\phi \, \partial_\mu \partial^\mu \phi \right) d^2x$$

$$= \int_{\partial\Sigma} 2\delta\phi \, \partial^\mu \phi \cdot n_\mu \, dr - \int_\Sigma 2\delta\phi \, \partial_\mu \partial^\mu \phi \, d^2x$$

$$= \int_{\partial\Sigma} 2\delta\phi \, \partial_n \phi - \int_\Sigma 2\delta\phi \, \partial_\mu \partial^\mu \phi \, dx \qquad (n_\mu) \text{ normal vector}$$

Since $\delta\phi$ is arbitrary, we have $\partial_\mu \partial^\mu \phi = 0$ (in the interior) and $\delta\phi \, \partial_n \phi = 0$ on $\partial\Sigma$

For general (M, g), the action S is given by

$$S = \int_\Sigma g_{ab} \partial_\mu \phi^a \partial^\mu \phi^b \, d^2x$$

We embed $(M, g) \to \mathbb{R}^K$ so that S can be expressed as

$$S = \sum_a \int_\Sigma \partial_\mu \phi^a \partial^\mu \phi^a \, d^2x$$

Similarly we get $0 = \delta S = \sum_a \left(\int_{\partial\Sigma} 2\delta\phi_a \partial_n \phi_a - \int_\Sigma 2\delta\phi_a \partial_\mu \partial^\mu \phi_a \right)$

with δ variation on M

Thus we have $(\Delta\phi^a)^{T_{\phi(x)}M} \equiv 0$ and $\delta\phi_a \partial_n \phi_a \equiv 0$

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We want to minimize

$$\int_\Sigma g_{ab}\, \partial_\mu \phi^a \partial^\mu \phi^b \, d^2x$$

∃ better way to write deformation

variation gives $(\psi = \delta\phi)$ ✓

$$0 = \frac{d}{dt}\Big|_{t=0} \int_\Sigma g_{ab}(\phi + t\psi)\, \partial_\mu(\phi^a + t\psi^a)\, \partial^\mu(\phi^b + t\psi^b)\, d^2x$$

$$= \int_\Sigma \partial_c g_{ab}\, \psi^c \partial_\mu \phi^a \partial^\mu \phi^b \, d^2x + 2\int_\Sigma g_{ab}\, \partial_\mu \phi^a \partial^\mu \psi^b \, d^2x$$

where $\int_\Sigma g_{ab}\, \partial_\mu \phi^a \partial^\mu \psi^b \, d^2x$

$$= -\int_\Sigma \left(\partial_c g_{ab}\, \psi^b \partial_\mu \phi^a \partial^\mu \phi^c - g_{ab}\, \psi^b \partial_\mu \partial^\mu \phi^a\right) d^2x$$

$$+ \int_{\partial\Sigma} g_{ab}\, \psi^b \partial_\mu \phi^a \cdot n_\mu \, dx$$

thus $\int_\Sigma g_{ab}\, \psi^b \partial_\mu \partial^\mu \phi^a \, d^2x$

$$= \int_\Sigma (\partial_c g_{ab} - \partial_a g_{bc} - \partial_b g_{ac})\, \psi^c \partial_\mu \phi^a \partial^\mu \phi^b \, d^2x$$

$$+ 2\int_{\partial\Sigma} \underbrace{g_{ab}\, \psi^b \partial_\mu \phi^a \cdot n_\mu}_{\partial_n \phi^a} \, dx$$

$$\Rightarrow \partial_\mu \partial^\mu \phi^i = \Gamma^i_{ab}\, \partial_\mu \phi^a \partial^\mu \phi^b$$

if we take $\psi^a = g^{ai} \mu_i$ for any function μ_i

and also $\delta\phi^b g_{ab}\, \partial_n \phi^a |_{\partial\Sigma} = 0$ ✓

SEASON