Calculus A (Fall 2010) Quiz 1

September 23, 2010

Dept.——————————— ID No.——————————— Name:———————————

Make sure to give sufficient reason in each problem or you will NOT get any credit for your answer.

A. (10 points) Consider the function $f(x) = x^3$ on $\mathbb{R}$ and a fixed point $a \in \mathbb{R}$. Given an $\varepsilon > 0$, find a $\delta > 0$ s.t. $|f(x) - f(a)| < \varepsilon$ whenever $|x - a| < \delta$.

B. (10 points) Suppose that f is continuous at a and $f(a) < 0$. Show that there exists an open interval I containing a s.t. $f(x) < 0$ for all $x \in I$.

C. (10 points) Show that the function $f(x) = \begin{cases} 0 & \text{if } x \text{ is irrational or } x = 0 \\ 1/q & \text{if } x = p/q \text{ rational in lowest terms} \end{cases}$ is discontinuous at $x = 1$.

Calculus A (Fall 2010) Quiz 2

September 30, 2010

Dept.————————————— ID No.————————————— Name:—————————————

Make sure to give sufficient reason in each problem or you will NOT get any credit for your answer.

A. (10 points) Consider a sequence $\{a_n\}_{n=1}^{\infty}$ defined by $\begin{cases} a_1 = \sqrt{3} \\ a_{n+1} = \sqrt{3 + a_n} \quad \text{for } n \geq 1 \end{cases}$. Show that $\lim_{n \to \infty} a_n$ exists and evaluate the limit.

B. (10 points) Show that $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \dfrac{1}{\sqrt{n^2+i}} = 1$.

C. (10 points) Let $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ be two sequence in $\mathbb{R}$. Suppose that $\lim\limits_{n\to\infty} a_n = a$ and $\lim\limits_{n\to\infty} b_n = b$, show that $\lim\limits_{n\to\infty} a_n b_n = ab$.

Calculus A (Fall 2010) Quiz 3

October 7, 2010

Dept._____ ID No._____ Name:_____

Make sure to give sufficient reason in each problem or you will NOT get any credit for your answer.

A. (10 points) Show that if every bounded sequence in $\mathbb{R}$ has at least one limit point, then every Cauchy sequence in $\mathbb{R}$ converges.

B. (10 points) Use the method of Riemann sum to show that $\int_a^b \cos x\, dx = \sin b - \sin a$.

C. (10 points) Suppose that f is a continuous function on a closed interval $[a, b]$. Show that f is uniformly continuous on $[a, b]$.

Calculus A (Fall 2010) Quiz 4

Dept._____   ID No._____   Name:_____

Make sure to give sufficient reason in each problem or you will NOT *get any credit for your answer.*

A. (10 points) Suppose that $f : \mathbb{R} \to \mathbb{R}$ is continuous and let $F(x) = \int_0^x f(t)dt$. Show that $F'(x) = f(x)$ for all $x \in \mathbb{R}$.

B. (10 points) Suppose that $f : \mathbb{R} \to \mathbb{R}$ is differentiable at $x = a$ and $f(a)$ is a maximum of f. Show that $f'(a) = 0$.

C. (10 points) Suppose that $f : \mathbb{R} \to \mathbb{R}$ is differentiable on $(a - \delta, a) \cup (a, a + \delta)$ for some $\delta > 0$ and $\lim\limits_{\substack{x \to a \\ x \neq a}} f'(x) = L$. Show that $f'(a)$ exists and equals to L.

Calculus A (Fall 2010) Quiz 5

October 21, 2010

Dept._____   ID No._____   Name:_____

A. (a) (5 points) State the Weierstrass principle in real number system.

    (b) (5 points) Use the Weierstrass principle to show that every bounded monotonic increasing sequence in $\mathbb{R}$ is convergent.

B. (10 points) Suppose $u(x) > 0$ and $v(x) > 1$ are two differentiable functions on $\mathbb{R}$. Find the derivative of the function $f(x) = \log_{v(x)} u(x)$ in terms of u, v, u' and v'.

C. (10 points) Let $f(x) = \sinh^{-1}(\cosh x)$, where $\sinh^{-1}$ is the inverse of hyperbolic sine function. Find $f'(x)$.

Calculus A (Fall 2010) Quiz 6

Dept._____ ID No._____ Name:_____

A. (5 points) Evaluate $\displaystyle\int_0^1 \sqrt{\frac{x}{1+x}}\,dx$.

B. Let $f(x) = \begin{cases} x^{2x} & \text{if } x > 0 \\ 1 & \text{if } x = 0 \end{cases}$.

(a) (5 points) Show that $\lim\limits_{\substack{x \to 0 \\ x \neq 0}} f(x)$ exists and f is continuous at $x = 0$.

(b) (5 points) Find all relative extrema of f.

(c) (5 points) Find all points of inflection of f if there is any.

C. Let $f(x) = \begin{cases} x^2 \sin \dfrac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$.

(a) (5 points) Show that f is differentiable at $x = 0$ and find $f'(0)$.

(b) (5 points) Find $f'(x)$ for all x. Is $f'(x)$ continuous?

Calculus A (Fall 2010) Quiz 7

Dept._____   ID No._____   Name:_____

A. (10 points) Evaluate $\displaystyle\int \frac{d\theta}{\cos\theta(1+\sin\theta)}dx$.

B. (10 points) Evaluate $\displaystyle\int_0^1 \frac{x^3}{\sqrt{x^2+1}}dx$.

C. (10 points) Evaluate $\displaystyle\int x^2 \tan^{-1} x\, dx$.