CALCULUS 2017: MIDTERM EXAM II

1. Use Riemann sum to calculate definite integrals:

$$(a) \quad \int_0^b x^2\,dx, \qquad (b) \quad \int_0^1 \sqrt{x}\,dx.$$

2. Calculate the following integrals:

$$(a) \quad \int x\tan^{-1} x\,dx, \qquad (b) \quad \int \sqrt{\frac{3x-1}{x^7}}\,dx.$$

3. Calculate the following integrals:

$$(a) \quad \int \frac{1}{x(x-1)^2}\,dx, \qquad (b) \quad \int \frac{dx}{\sqrt{x^2-1}}.$$

4. Let $S_n(x) = \int \sin^n x\,dx$, where $n = 0, 1, 2, \ldots$.
(a) Derive the recursive relation:

$$S_n(x) = -\frac{1}{n}\sin^{n-1} x\cos x + \frac{n-1}{n} S_{n-2}(x), \qquad n \geq 2.$$

(b) Derive a formula of $\int_0^{\pi/2} \sin^{2k} x\,dx$ for all $k \in \mathbb{N}$.

5. Decide whether the following improper integrals exist:

$$(a) \quad \int_{-\infty}^{\infty} e^{-x^2}\,dx, \qquad (b) \quad \int_{-\infty}^{\infty} \frac{1}{x^3+x-1}\,dx.$$

6. Consider the ellipse on the x, y plane:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

with $a, b > 0$. Calculate
(a) the area of the upper half region Ω (with $y \geq 0$) inside the ellipse,
(b) the volume of the solid B arising from the rotation of Ω in the x-axis,
(c) the center of mass $(\bar{x}, \bar{y})$ of Ω.

7. (a) Prove the Fundamental Theorem of Calculus for integrals $\int_a^x f(t)\,dt$ where f is a continuous function. (b) Compute

$$\frac{d}{dx}\int_{g(x)}^{h(x)} f(t)\,dt.$$

Date: December 7, 2017, pm 3:30 – 6:30. Each problem is of 15 points. You may work on each part separately by assuming the other parts. (A course by Chin-Lung Wang at NTU.) .