2018 ALGEBRA QUIZ: I

1. Let G be a semigroup with a right unit 1_r and every $a \in G$ has a right inverse b relative to 1_r, i.e. $ab = 1_r$. Show that G is a group.

2. Let G be a finitely generated group and H a subgroup of finite index. Show that H is finitely generated.

3. Show that S_n is generated by (12) and $(12\ldots n)$.

4. Let p be the smallest prime dividing the order of a finite group G. Show that any subgroup H of G of index p is normal.

5. Assume Sylow I, prove Sylow II. That is, show that (a) any p subgroup is contained in a Sylow p group and any two Sylow p groups are conjugate, (b) $|\mathrm{Syl}_p(G)| \equiv 1 \pmod{p}$ and dividing $|G|/p^k$ (where $p^k \| |G|$).

Show your answers/computations/proofs in details. Date: pm 2:20 – 3:20, October 11, 2018 at AMB 101. A course by Chin-Lung Wang at NTU..