2019 ALGEBRA II - MIDTERM EXAM

1. Let R and R' be Morita similar. Show that R is primitive if and only if R' is. Also in any correspondence between ideals, $J(R)$ corresponds to $J(R')$.

2. Show that a ring R is simple left artinian $\Longleftrightarrow R \cong M_n(\Delta)$ where Δ is a division ring, $n \in \mathbb{N}$.

3. Prove Frobenius' theorem on division rings over $\mathbb{R}$ or Wedderburn's theorem on finite division rings. You can use any method you know.

4. Let ρ, ρ' be representations of G over an infinite field F. If $\rho \otimes_F K \cong \rho' \otimes_F K$ for an extension field K of F, show that $\rho \cong \rho'$. How about if $|F| < \infty$?

5. Let $H \subset K \subset G$ be subgroups with G finite, σ and ρ are complex representations of H and G respectively. Using character calculations to prove (1) $(\sigma^K)^G \cong \sigma^G$, (2) $\sigma^G \otimes \rho \cong (\sigma \otimes \rho_H)^G$, (3) $(\sigma^G)^* \cong (\sigma^*)^G$.

6. Show that the quaternion group Q_8 is not isomorphic to D_4, but they have the same character table. Describe the rings $F[Q_8]$ and $F[D_4]$ for $F = \mathbb{Q}, \mathbb{C}$.

7. Construct the character table for $G = A_5$. (You get partial credits for doing the simpler case $G = S_4$.)

8. Present an essential topic/theorem in modules, rings, or representations that you have well-prepared but not shown in the above problems.

Each problem is of 15 points (total 120 pts). Be sure to show your answers/computations/proofs in details. Time: pm 5:30 – 9:30, April 22, 2019 at AMB 101. A course by Chin-Lung Wang at NTU..