

LECTURES ON CALABI-YAU THREEFOLDS

— Deformations and Special Lagrangian Geometry —

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Part I. Deformation Theory of Calabi-Yau Threefolds

ABSTRACT

The Bogomolov-Tian-Todorov theorem states that the deformation theory of a Calabi-Yau manifold is well-behaved: every infinitesimal deformation can be lifted to an actual deformation. I will discuss generalisations of this theorem to the case of singular Calabi-Yau threefolds with varying degrees of singularities. These results will then be applied to understand smoothability of Calabi-Yau threefolds with various singularities. This is of special interest in light of Miles Reid's idea of connecting together all moduli spaces of Calabi-Yau threefolds via a process of deformation and resolution.

LECTURE 1: 1999 1/1 (Friday), pm 1:00 – 3:00 (Academia Sinica)

LECTURE 2: 1999 1/2 (Saturday), am 10:00 – 12:00 (Academia Sinica)

Part II. Special Lagrangian Geometry

ABSTRACT

These talks will concern my investigations into the Strominger-Yau-Zaslow mirror symmetry conjecture. A special Lagrangian submanifold of a Calabi-Yau manifold is a real submanifold of half dimension which is both Lagrangian with respect to the Kaehler form and for which the imaginary part of the holomorphic n -form restricts to zero. These form an important class of minimal submanifolds of Calabi-Yau manifolds.

The SYZ conjecture suggests that mirror symmetry arises through the existence of so called special Lagrangian torus fibrations, $f : X \rightarrow B$ from a Calabi-Yau manifold X whose fibres are, in general, special Lagrangian tori. The mirror of X is then constructed by dualising the fibration in a well-defined sense.

I will discuss how this gives rise to the usual interchange of Hodge numbers in three-dimensional mirror symmetry, methods of placing complex and Kaehler structures on the dual fibration, and end with a complete working out of the details in the K3 case, where everything can be verified. In particular, this produces a version of mirror symmetry for K3 surfaces which does not require the classification of K3 surfaces.

LECTURE 3: 1999 1/4 (Monday), pm 1:00 – 3:00 (Academia Sinica)

LECTURE 4: 1999 1/8 (Friday), pm 1:00 – 3:00 (Taiwan University)

LECTURE 5: 1999 1/9 (Saturday), am 10:00 – 12:00 (Taiwan University)

LECTURE 6: 1999 1/11 (Monday), pm: 1:00 – 3:00 (Taiwan University)

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