Imaging Sciences and Mathematics

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Fall 2010

Imaging Sciences

- The SIAM *Journal on Imaging Sciences* covers all areas of imaging sciences, broadly interpreted. It includes
  - image formation (imaging)
  - image processing
  - image analysis
  - image interpretation and understanding
  - computer graphics and visualization
  - inverse problems in imaging;

- leading to applications to diverse areas in science, medicine, engineering, and other fields.

Special Thanks to Raymond Chan and Chiu-Yen Kao for their slides.

Imaging Sciences

- Image Acquistion (Imaging)

  - human vision, Optics, Radar imaging, Ultrasound, MRI, X-ray CT,…

- Image Processing

$$I_{input} \xrightarrow{T} I_{output} = T[I_{input}]$$

- Image Interpretation (Visual Intelligence)

Image Processing

- What is Image?
- What is Image Enhancement?
  - Contrast Enhencement
  - Image Denoising
  - Image Deblurring
- Image Inpainting
- Image segmentation
- Image Registration

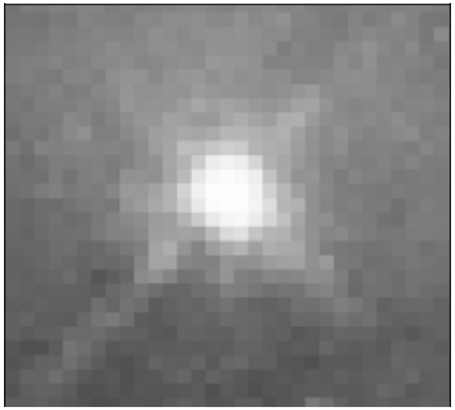
Book: Rafael C. Gonzalez and Richard E. Woods,
Digital Image Processing, Prentice Hall

What are Digital Images?

1. What is a digital image?

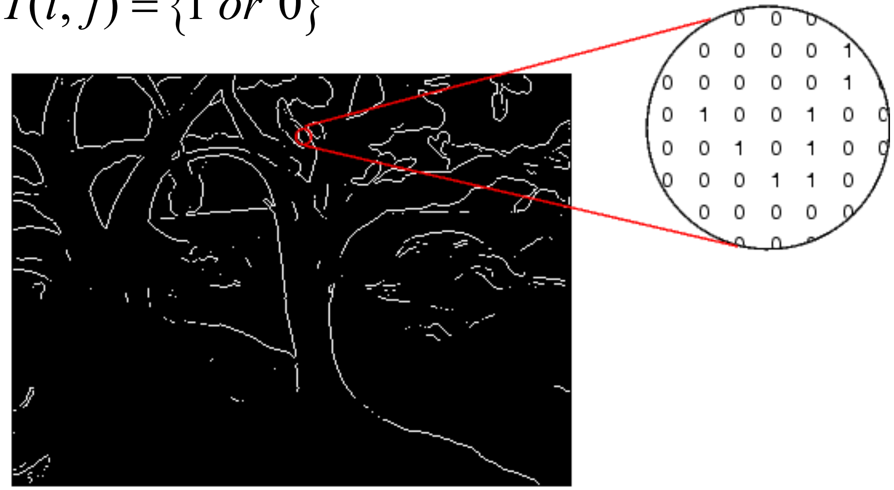
$$I : \Omega \rightarrow R \xrightarrow{\text{sampling, quantized}} I_d : \{1 \le i \le m, 1 \le j \le n\} \rightarrow R_k, 1 \le k \le l$$

A digital image Is an array, or a matrix , of square pixels (picture elements) arranged in columns and rows.



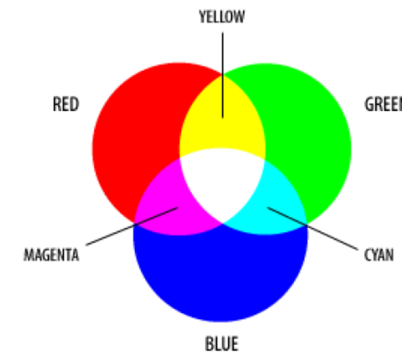
a. Binary Image (logical array)

$$I(i, j) = \{1 \; or \; 0\}$$



Chiu-Yen Kao

What are Digital Images?



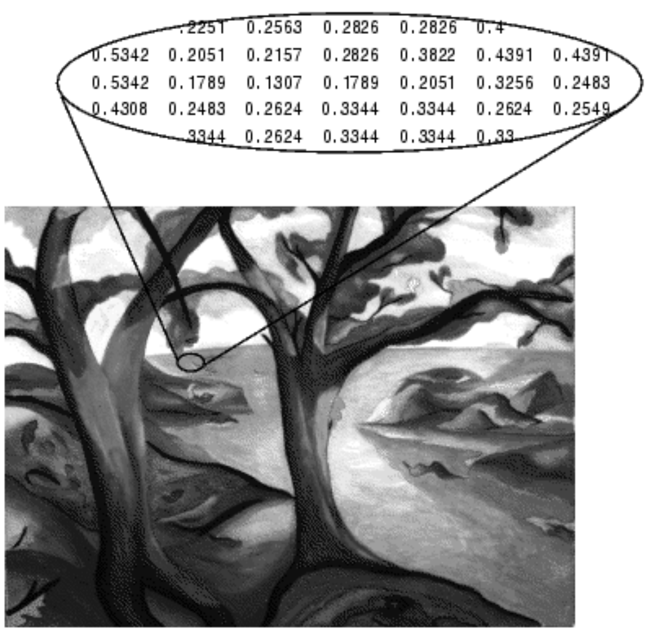
b. **Intensity Image**

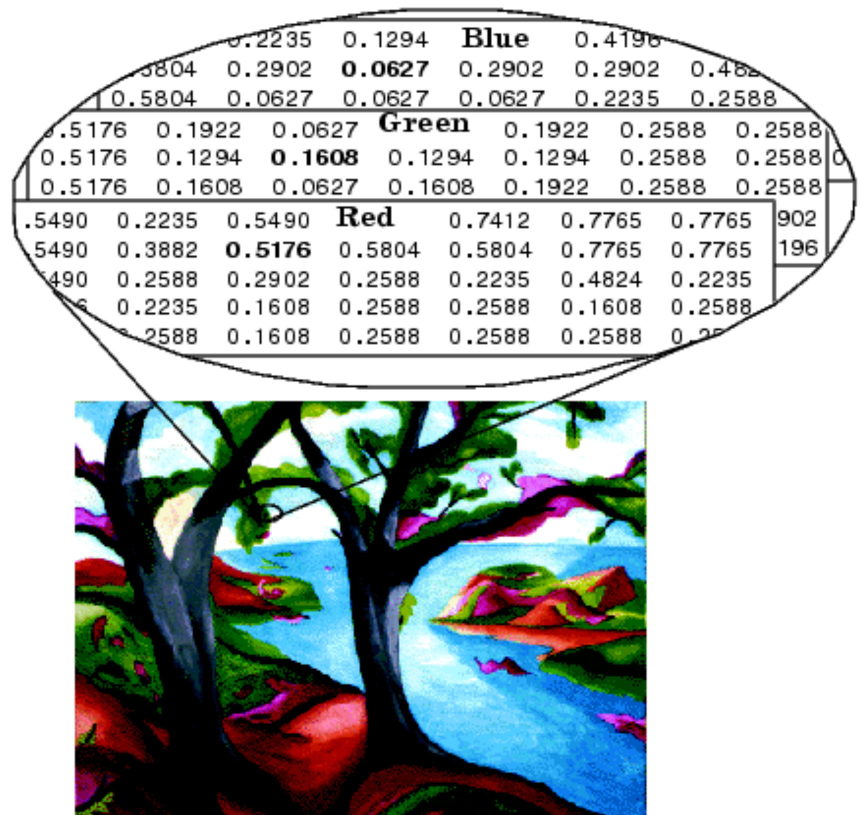
8 bit (uint8, 0-255), 16 bit (uint16, 0-65535) and double ([0 1])

c. **color Image**

RGB:

24 bit = 256^3 ~ 16 million colors

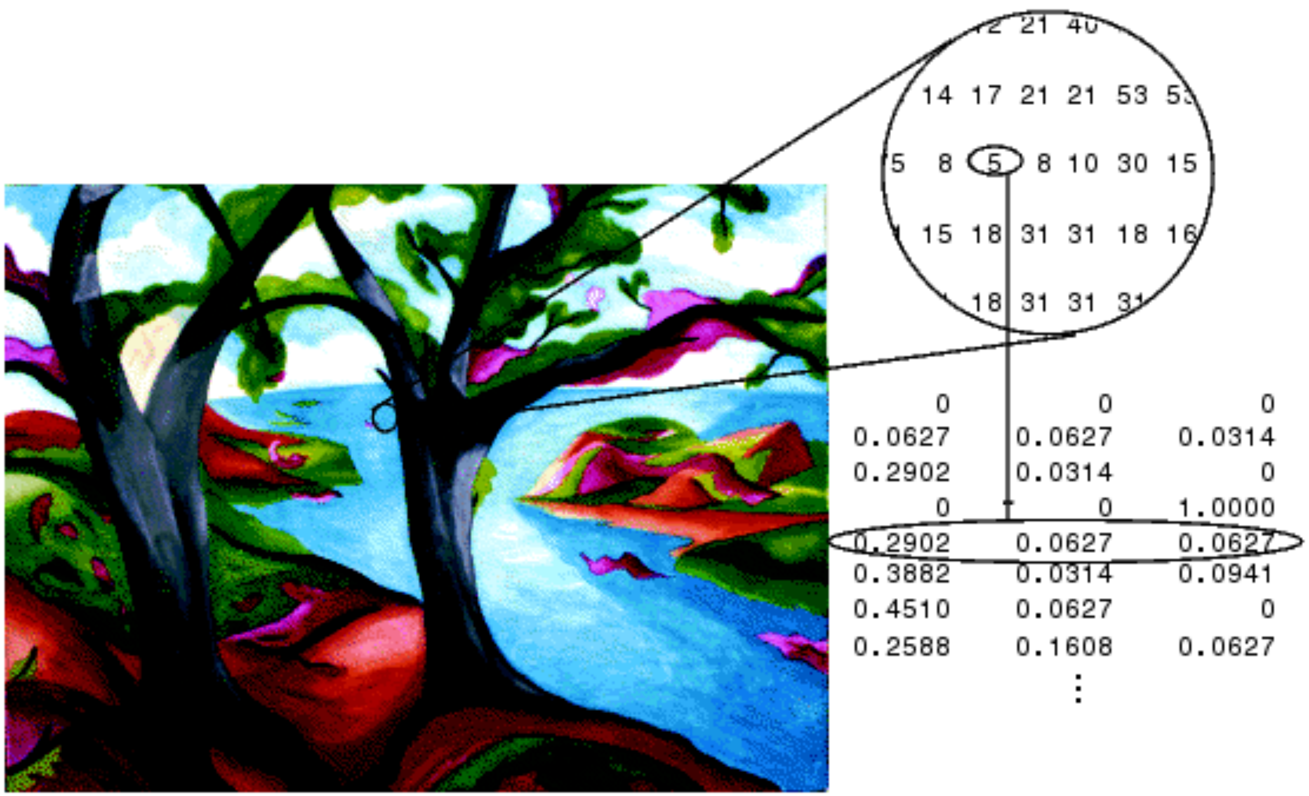


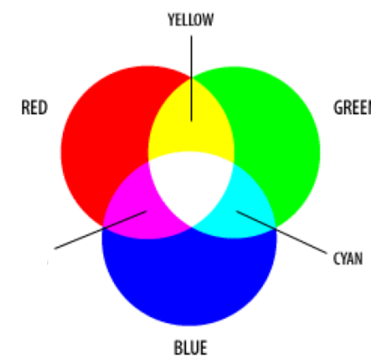


What are Digital Images?

c. index color Image

data matrix and colormap matrix





Chiu-Yen Kao

Examples of images

- Daily-life images
- Astro images
- Medical images

Standard images

No higher resolution available.

Lenna.png (512 × 512 pixels, file size: 464 KB, MIME type: image/png)

Hubble site

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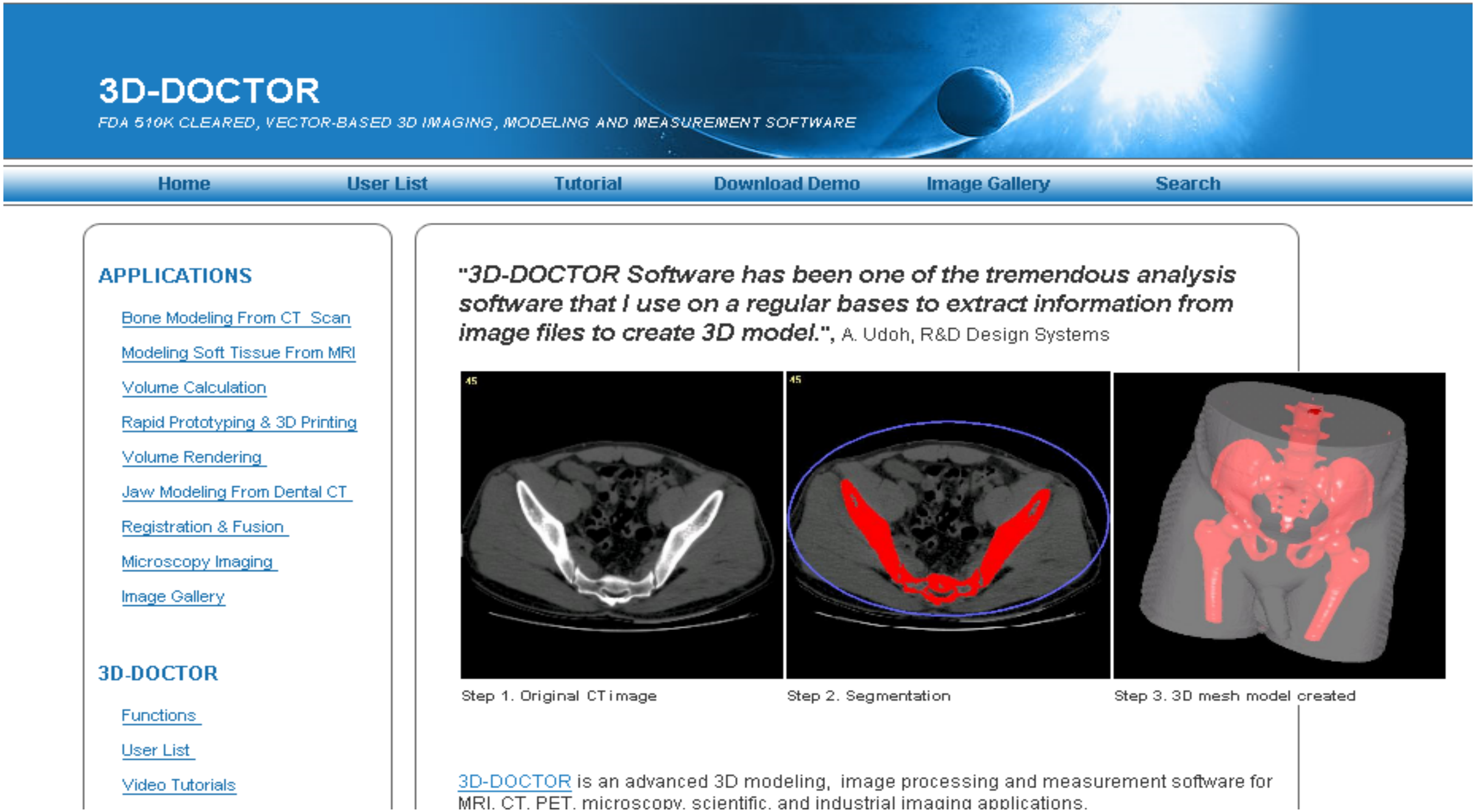
MOVIE THEATER
IMAGE TOURS
WALL MURALS
New Hubble Images
PICTURE ALBUM
View an expansive collection of Hubble images: planets, stars, nebulae, galaxies and more. Picture Album spreads the universe out before you in all its dazzling complexity.
WALLPAPER
Fill your computer screen with a Hubble image. Your body may be trapped at your desk, but your imagination can roam the far reaches of the universe with our cosmic visuals.
IMAGE TOURS
Take a brief tour of some of Hubble's photos. We'll point out what's in the images and give you some fast facts about them.
Take our latest tour of this brilliant object!
Star V838 Monocerotis
WALL MURALS
Hubble images become large-scale artwork for your home. Have these

3D-Doctor



3D-DOCTOR
FDA 510K CLEARED, VECTOR-BASED 3D IMAGING, MODELING AND MEASUREMENT SOFTWARE
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"3D-DOCTOR Software has been one of the tremendous analysis software that I use on a regular bases to extract information from image files to create 3D model.", A. Udoh, R&D Design Systems
Step 1. Original CT image
Step 2. Segmentation
Step 3. 3D mesh model created
3D-DOCTOR is an advanced 3D modeling, image processing and measurement software for MRI, CT, PET, microscopy, scientific, and industrial imaging applications.

Image Processing

- What is Image?
- What is Image Enhancement?
  - Contrast Enhencement
  - Image Denoising
  - Image Deblurring
- Image Inpainting
- Image segmentation
- Image Registration

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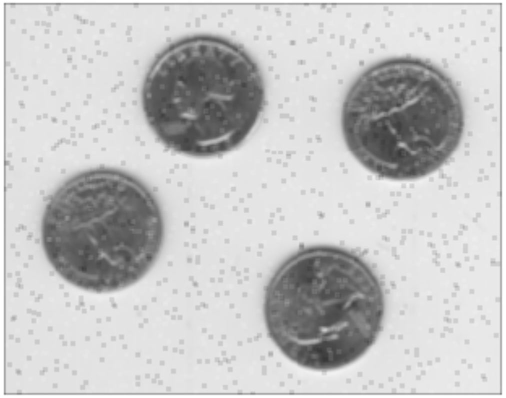
Image Enhancement

1. **Image Enhancement**

a. **Intensity Adjustment**


b. **Denoise**




c. **Deblur**


Chiu-Yen Kao

Image Inpainting

"Image Inpainting : An Overview", Guillermo Sapiro

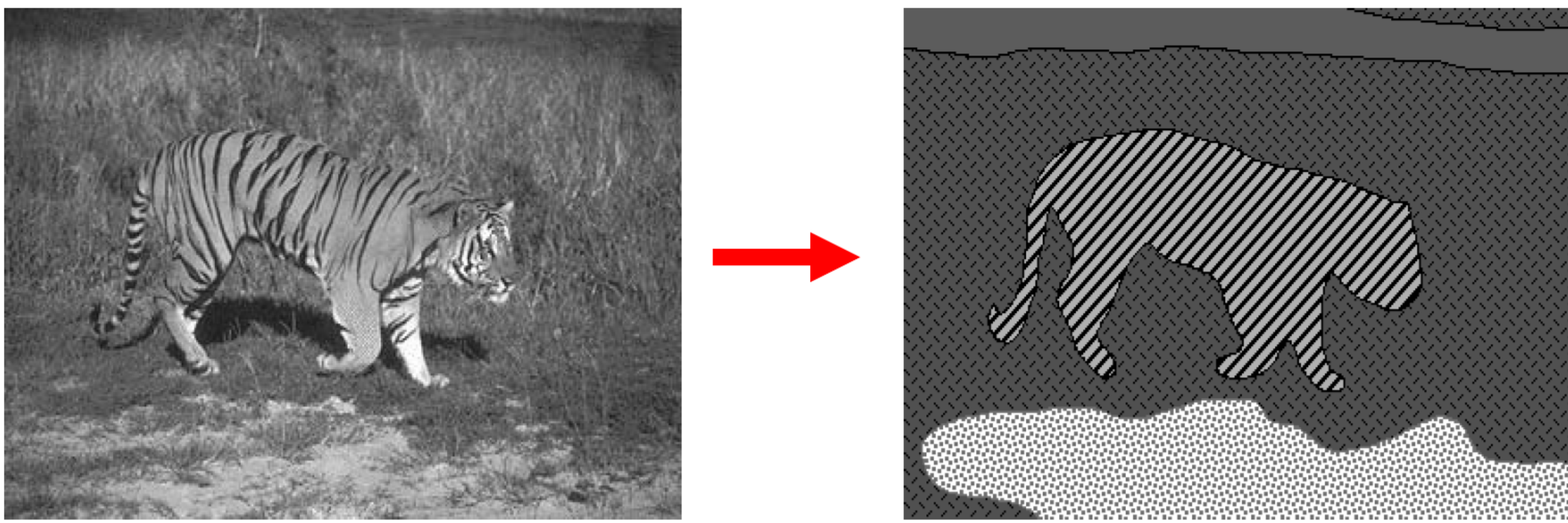
"Fast Digital Image Inpainting", Manuel M. Oliveira, Brian Bowen, Richard McKenna and Yu-Sung Chang

Chiu-Yen Kao

Introduction to Image Segmentation

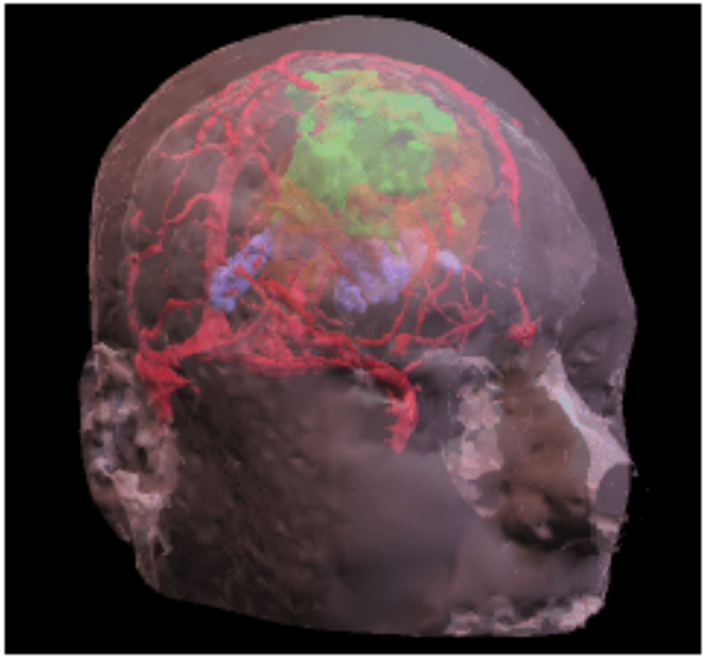
Chiu-Yen Kao

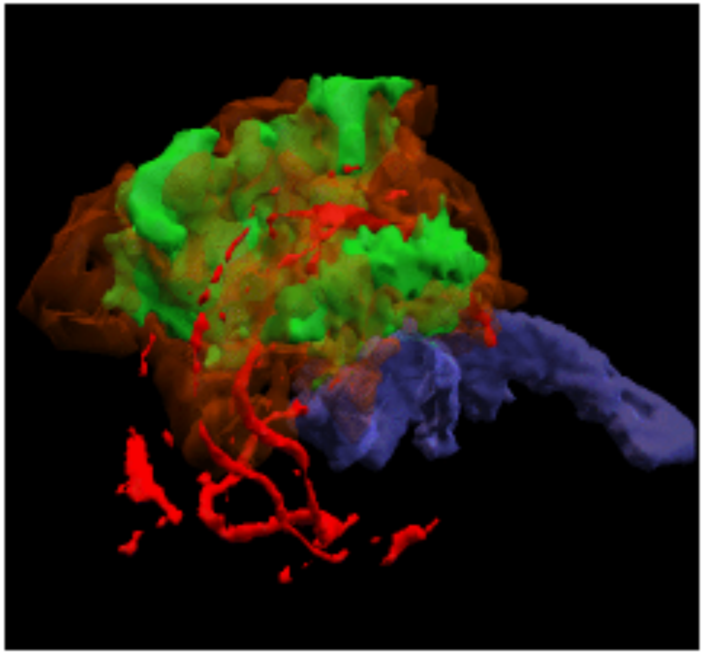
$$X = \bigcup_{i=1}^{N} R_i,\ R_i \cap R_j = 0 \ \ for\ i \neq j$$



Chiu-Yen Kao

Image Registration





Tumor(green), Vessels(red), Ventricles(blue), Edema (orange)

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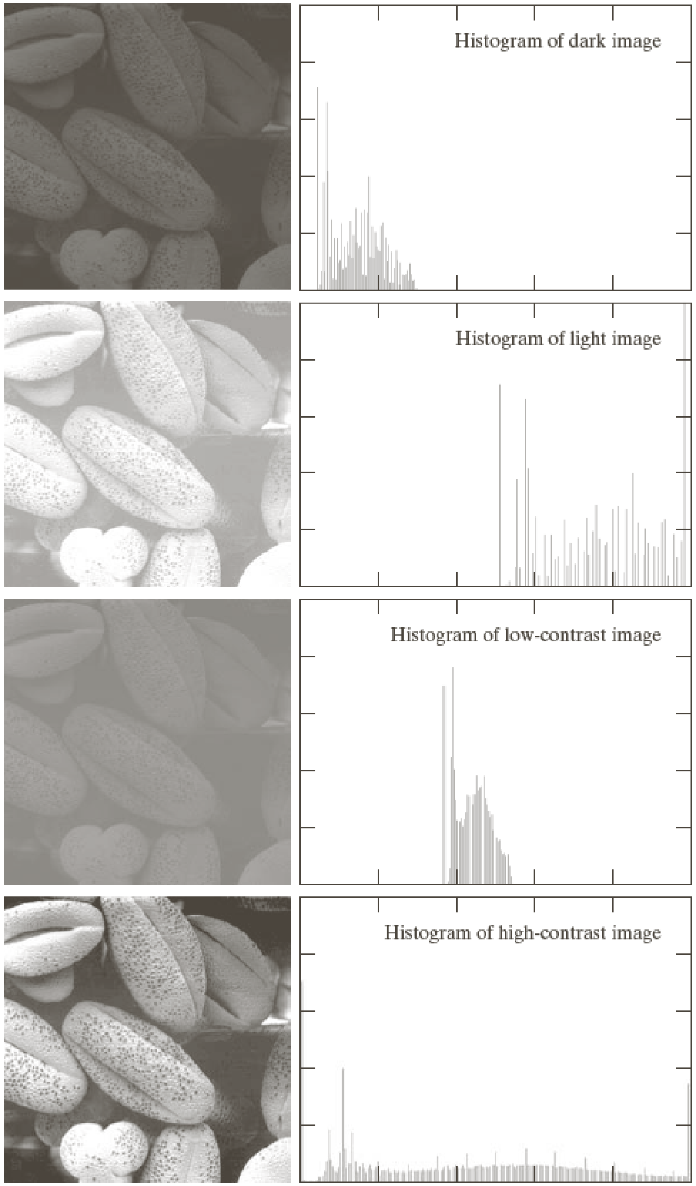
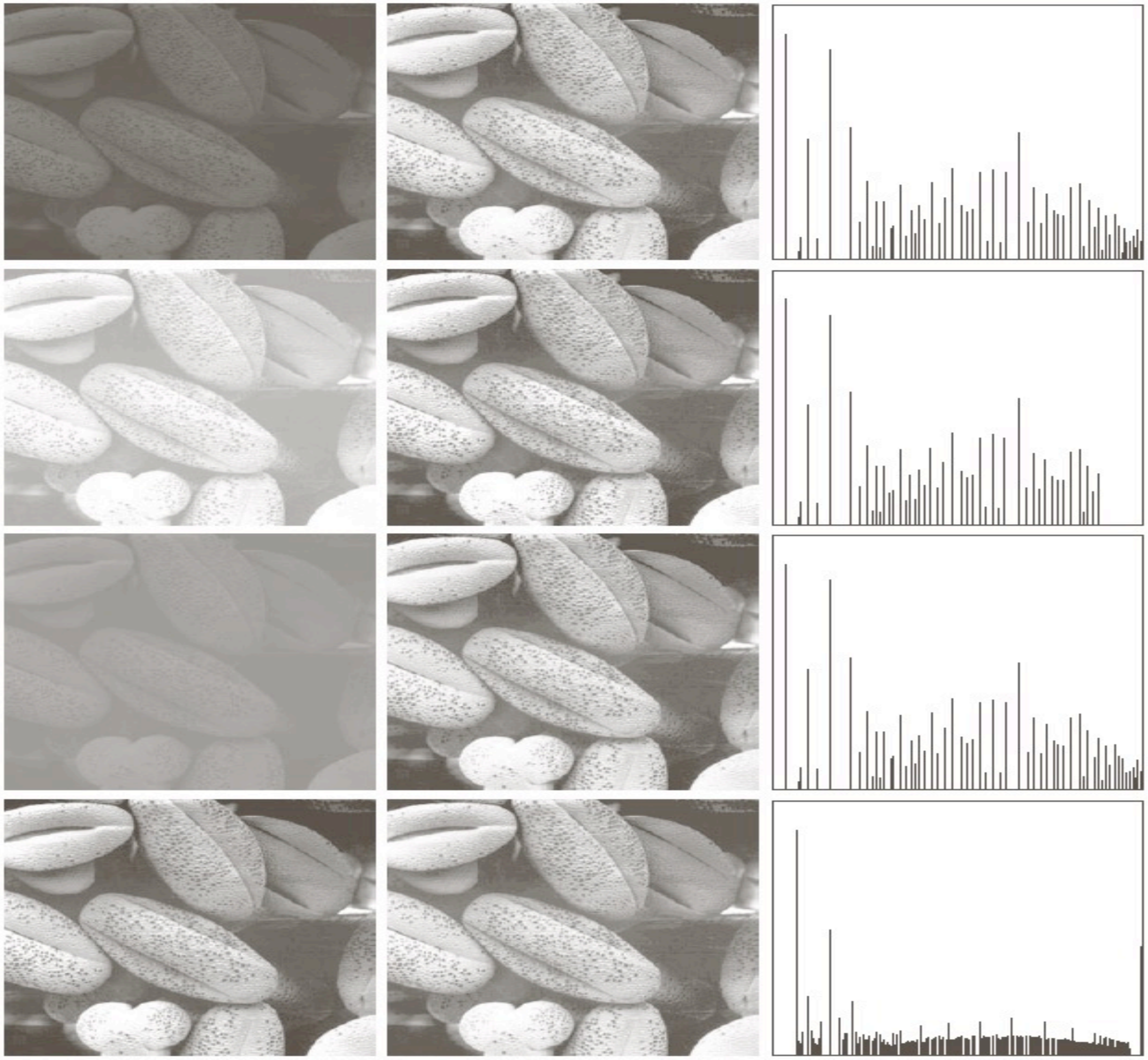
Contrast enhancement-1



Histogram

Contrast enhancement-2

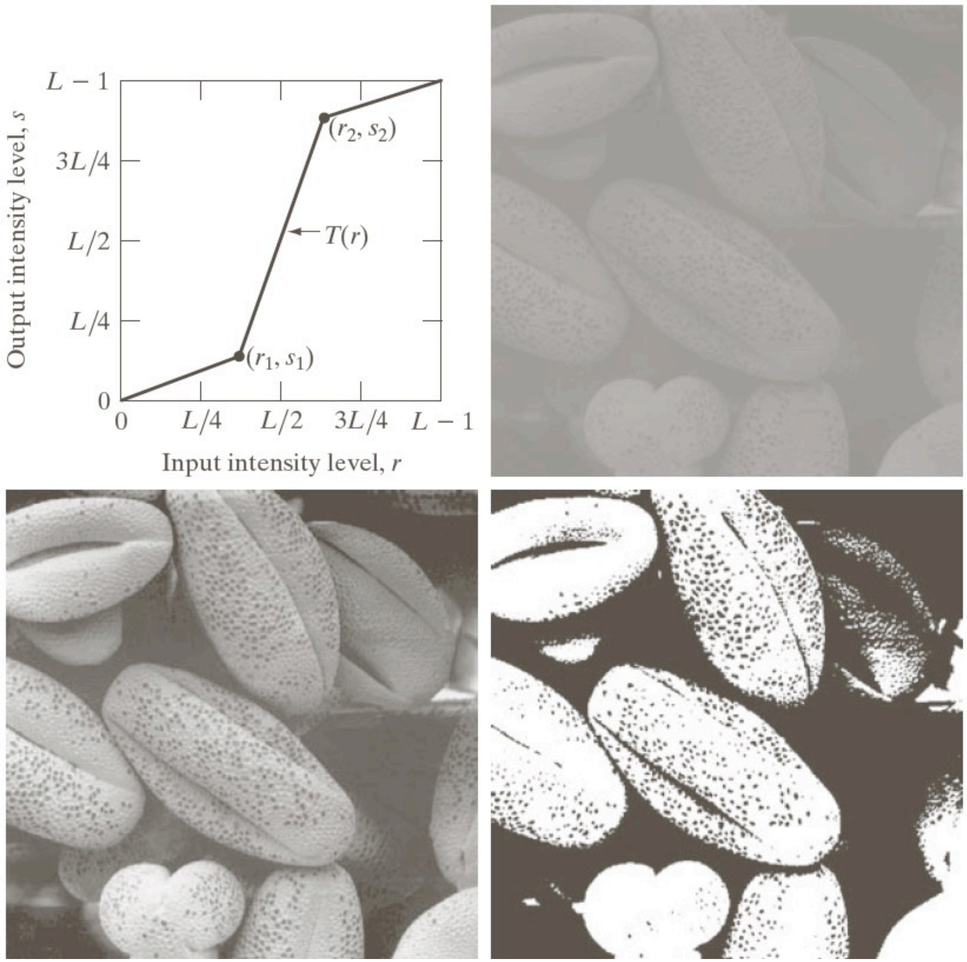


Histogram equalization

$$g(x, y) = T[f(x, y)]$$

Contrast Enhancement -3

- Gray-level transform $g(x, y) = T[f(x, y)]$



a b
c d

FIGURE 3.10 Contrast stretching. (a) Form of transformation function. (b) A low-contrast image. (c) Result of contrast stretching. (d) Result of thresholding. (Original image courtesy of Dr. Roger Heady, Research School of Biological Sciences, Australian National University, Canberra, Australia.)

Image Processing

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Noise models

- Assume white noise

$$n(x, y) \text{ and } n(x', y') \text{ are uncorrelated}$$

- Types of noises
  - Additive noise

$$g = f + n, \; n: \text{mean } 0, \text{ variance } \sigma^2$$

  - Multiplicative noise

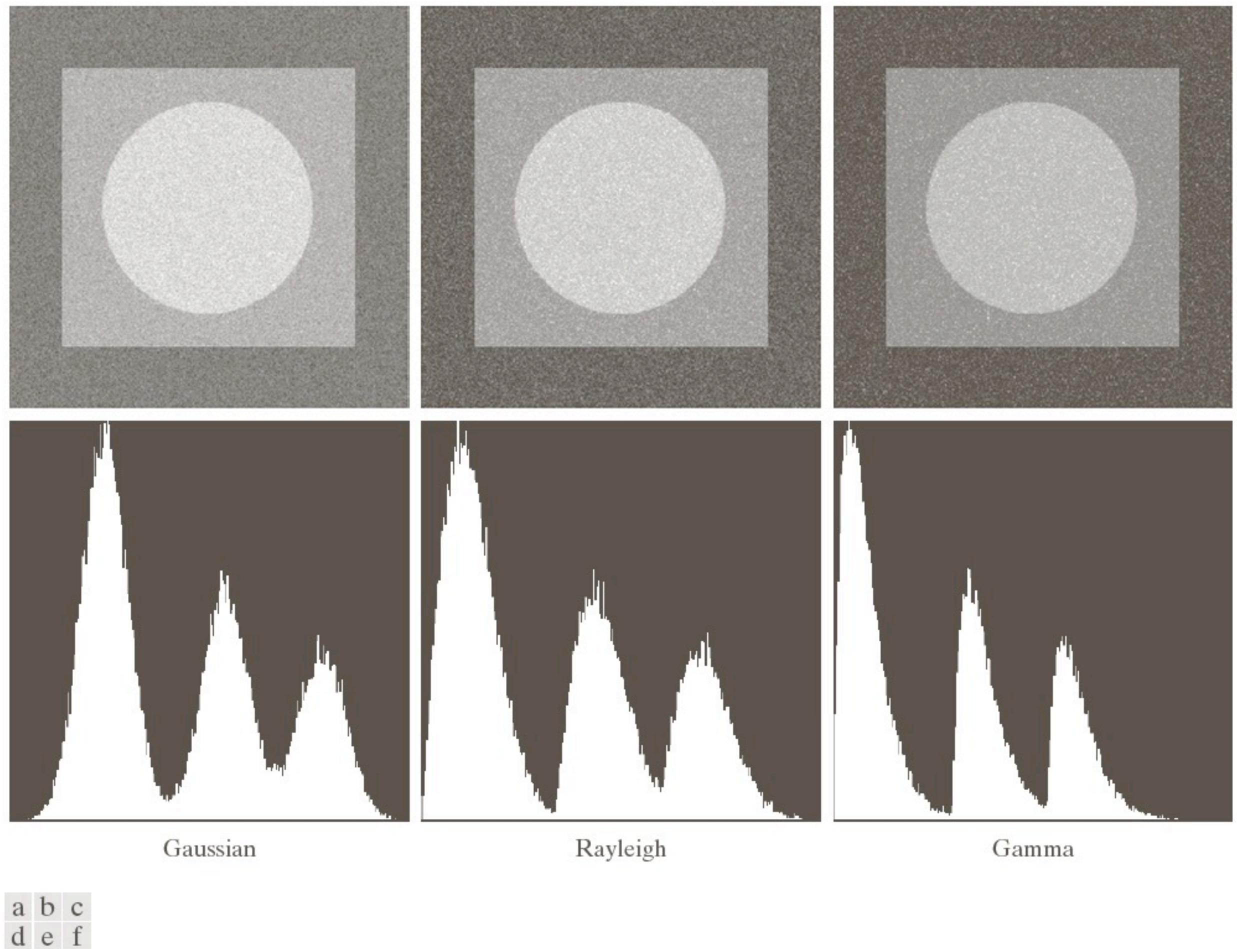
$$g = fn, \; n: \text{mean } 1, \text{ variance } \sigma^2$$

  - Mixed

$$g = fn_1 + n_2$$

Noise Models-2

Gaussian Rayleigh Gamma



$$g = f + n$$

FIGURE 5.4 Images and histograms resulting from adding Gaussian, Rayleigh, and gamma noise to the image in Fig. 5.3.

Noise Models-3

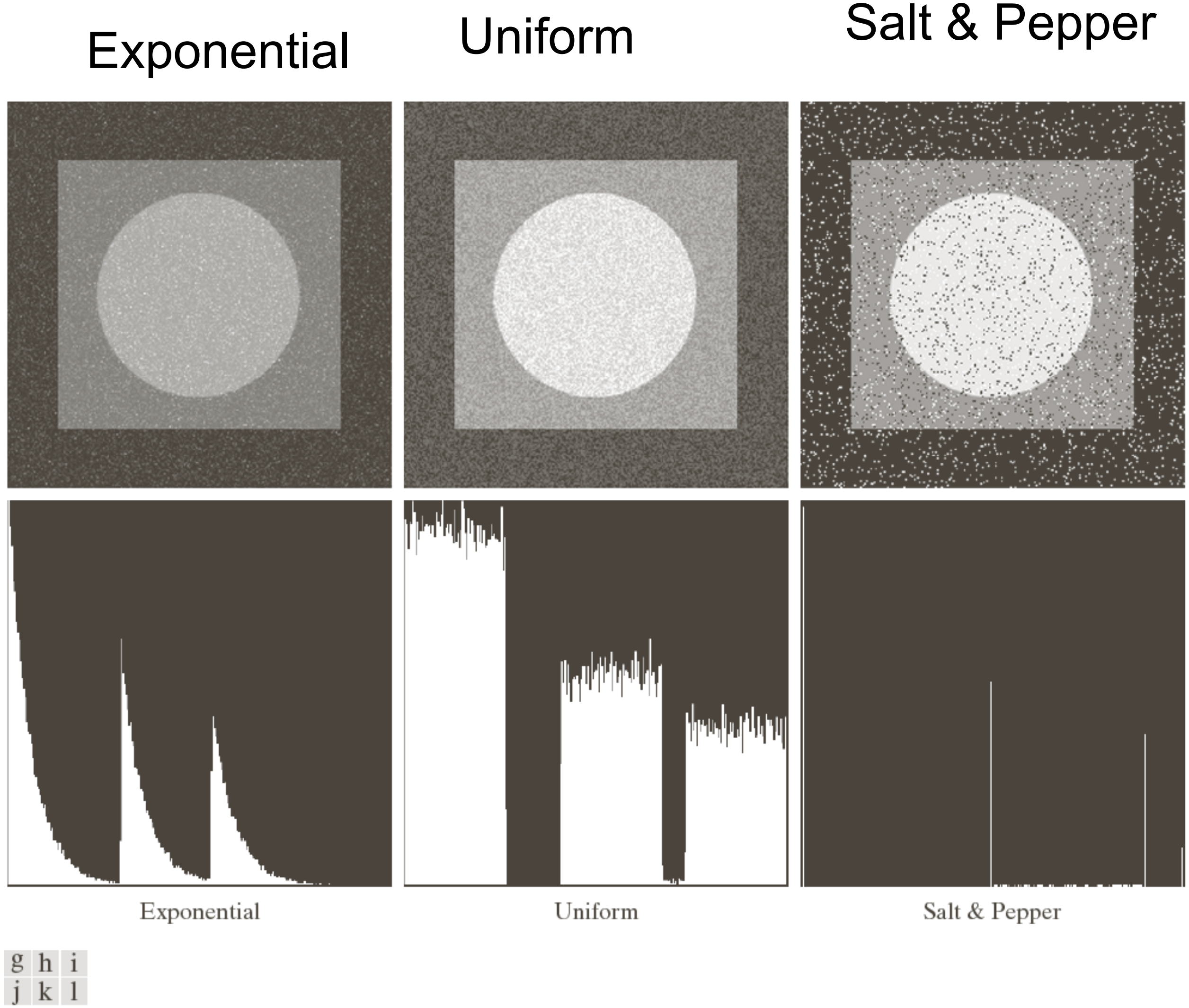


FIGURE 5.4 *(Continued)* Images and histograms resulting from adding exponential, uniform, and salt and pepper noise to the image in Fig. 5.3.

Denoise methods

- Filtering techniques
  - Spatial filtering
    - Mean filters
    - Order-Statics filters
  - Frequency filtering
  - Wavelet filtering
- Variational approach

Spatial filtering

- Mean filters:
  - Arithmetic mean filter

$$g = f + n$$

$$\tilde{f}(x,y) = \frac{1}{mn} \sum_{(s,t)\in S_{x,y}} g(s,t)$$

  - Geometric mean filter

$$g = fn$$

$$\tilde{f}(x,y) = \left[\prod_{(s,t)\in S_{x,y}} g(s,t) \right]^{\frac{1}{mn}}$$

  - Harmonic mean filter

$$\tilde{f}(x,y) = \frac{mn}{\sum_{(s,t)\in S_{x,y}} \frac{1}{g(s,t)}}$$

Mean filterings

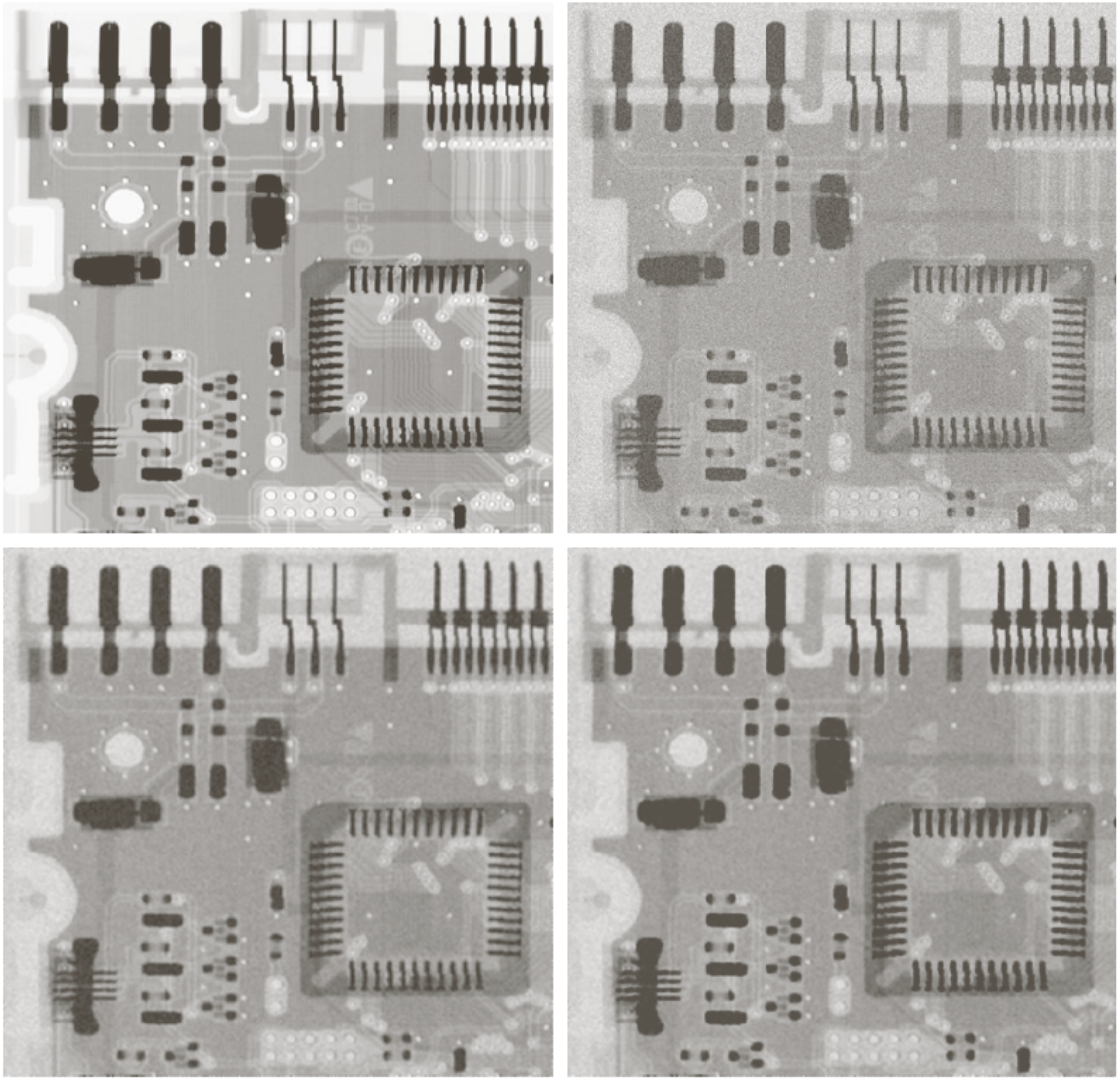


FIGURE 5.7 (a) X-ray image. (b) Image corrupted by additive Gaussian noise. (c) Result of filtering with an arithmetic mean filter of size 3×3. (d) Result of filtering with a geometric mean filter of the same size. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

Mean filtering

- Convolution with a smoothing mask

$$\tilde{f}_{i,j} = h * g := \sum_{|s|,|t| \leq 1} h_{s,t} g_{i-s,j-t}$$

$$\frac{1}{16}$$

1	2	1
2	4	2
1	2	1

$h_{s,t}$

Denoise methods

- Filtering techniques
  - Spatial filtering
    - Mean filters
    - Order-Statics filters
  - Frequency filtering
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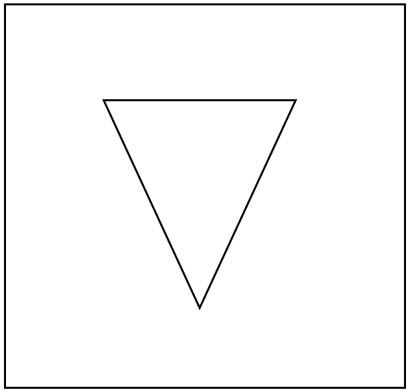
- Variational approach

Impulse Noise Model



Original Image
(a triangle)

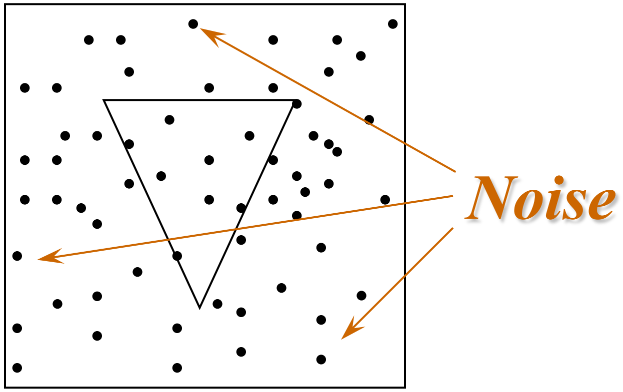


Image corrupted
by *Impulse Noise*

Only a number of pixels are corrupted

Impulse Noise Model

Impulse Noise are caused by

- **Malfunctioning pixels** in camera sensors
- **Faulty memory locations** in hardware
- **Transmission** in a noisy channel

Two types of Impulse Noise

I. Salt-and-Pepper Noise

II. Uniformly-Distributed Random Noise

Salt-and-Pepper Noise

$\mathbf{x} = (x_{i,j})$: true image with $x_{i,j} \in [0, 255]$.
$\mathbf{y} = (y_{i,j})$: observed noisy image.

$$y_{i,j} = \begin{cases} 0 & \text{with probability } r/2\%, \\ 255 & \text{with probability } r/2\%, \\ x_{i,j} & \text{with probability } 1 - r\%. \end{cases}$$

Noise level $= r\%$.

Noise-free Image

At 10% Noise

At 30% Noise

At 50% Noise

Random-Valued Impulse Noise

$\mathbf{x} = (x_{i,j})$: true image with $x_{i,j} \in [0, 255]$.
$\mathbf{y} = (y_{i,j})$: observed noisy image.

$$y_{i,j} = \begin{cases} n_{i,j} & \text{with probability } r, \\ x_{i,j} & \text{with probability } 1 - r, \end{cases}$$

where $n_{i,j}$ is randomly distributed in [0,255].

Denoising Schemes

Median Filter

Noisy Image

$y_{i-1,j-1}$	$y_{i-1,j}$	$y_{i-1,j+1}$
$y_{i,j-1}$	$y_{i,j}$	$y_{i,j+1}$
$y_{i+1,j-1}$	$y_{i+1,j}$	$y_{i+1,j+1}$

Restored Image

$y_{i-1,j-1}$	$y_{i-1,j}$	$y_{i-1,j+1}$
$y_{i,j-1}$	y_{i_5}	$y_{i,j+1}$
$y_{i+1,j-1}$	$y_{i+1,j}$	$y_{i+1,j+1}$

Sort

Recovered

$$y_{i_1} \leq y_{i_2} \leq y_{i_3} \leq y_{i_4} \leq y_{i_5} \leq y_{i_6} \leq y_{i_7} \leq y_{i_8} \leq y_{i_9}$$

Median

30% Salt-and-Pepper Noise

Median filter

Median-type Filters

- Drawback of Median Filter: Every pixel is modified, hence fuzziness and blurring
- Extensions of Median Filters (Median-type Filters):
  - Adaptive Median Filter (Wang, *IEEE Trans IP, (1995))*
  - Adaptive Center Weighted Median Filter (2001)
  - Multi-state Median Filters (2001)
  - Filter based on homogeneity info (2003)
  - …
  - Detection statistics (*IEEE TIP* 2007)

Adaptive Median Filter

Noisy Image

$y_{i-1,j-1}$	$y_{i-1,j}$	$y_{i-1,j+1}$
$y_{i,j-1}$	$y_{i,j}$	$y_{i,j+1}$
$y_{i+1,j-1}$	$y_{i+1,j}$	$y_{i+1,j+1}$

Sort

$$y_{i_1} \leq y_{i_2} \leq y_{i_3} \leq y_{i_4} \leq y_{i_5} \leq y_{i_6} \leq y_{i_7} \leq y_{i_8} \leq y_{i_9}$$

Median

If **Median** $= y_{i_1}$ or y_{i_9}, then increase window size.

Characteristics of Median-type Filters

Two Steps

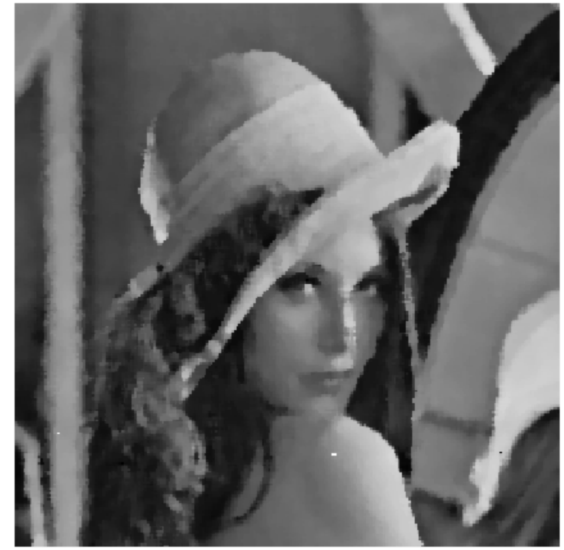
1. ***Noise Detection*** (e.g., thresholding)
2. ***Noise Replacement*** (by **Median** or its variants)

Advantages

1. ***Fast***
2. ***Accurate Detection***

30% Salt-and-Pepper Noise

Median Filter



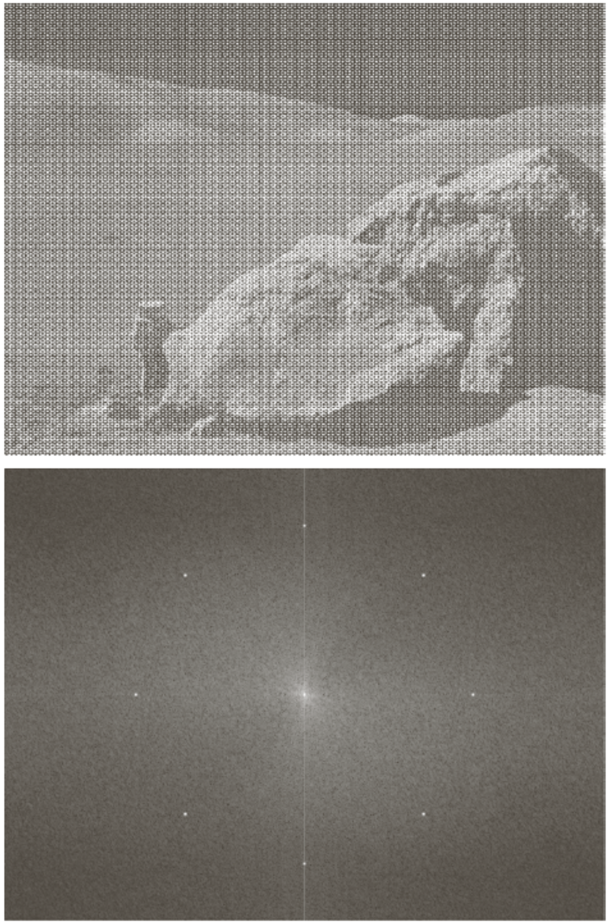
Adaptive Median Filter

Denoise methods

- Filtering techniques
  - Spatial filtering
    - Mean filters
    - Order-Statics filters
  - Frequency filtering
- Variational approach

Frequency filter

- Noise in frequency



a
b

FIGURE 5.5
(a) Image corrupted by sinusoidal noise. (b) Spectrum (each pair of conjugate impulses corresponds to one sine wave). (Original image courtesy of NASA.)

Frequency filtering

- Taking Fourier transform:

$$\hat{f}(\xi,\eta) = \iint f(x,y)e^{-i(x\xi+y\eta)}dxdy$$

- Noise model:

$$\hat{g} = \hat{f} + N$$

- Band reject/pass filter

$$\hat{\tilde{f}}(\xi,\eta) = k(\xi,\eta)\hat{g}(\xi,\eta)$$

Bandreject filter

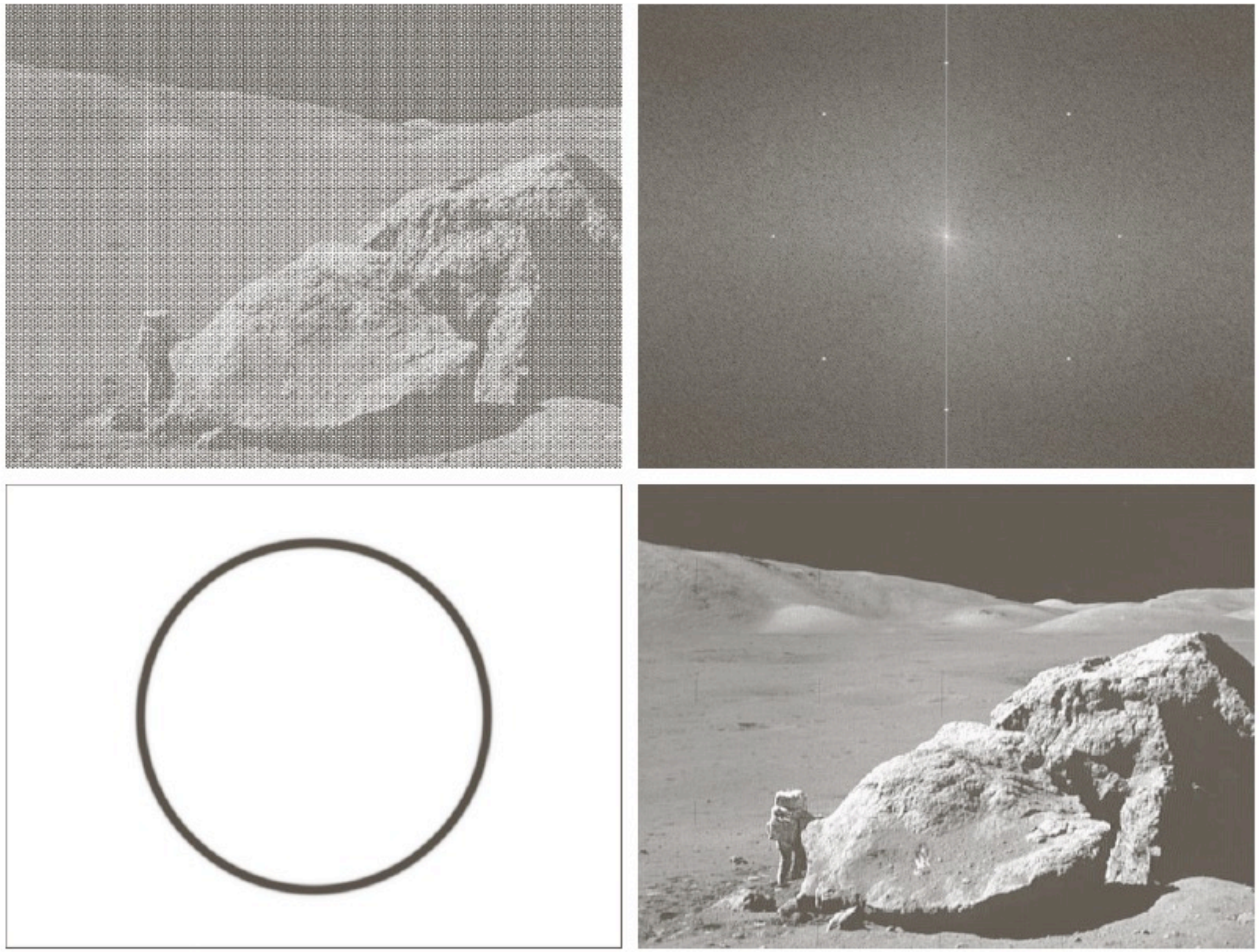


FIGURE 5.16 (a) Image corrupted by sinusoidal noise. (b) Spectrum of (a). (c) Butterworth bandreject filter (white represents 1). (d) Result of filtering. (Original image courtesy of NASA.)

Denoise methods

- Filtering techniques
  - Spatial filtering
    - Mean filters
    - Order-Statics filters
  - Frequency filtering
  - Wavelet filtering
- Variational approach

Variation approach-1

- Noise model: $z = u + n$ $\quad n:$ mean 0, variance σ^2

- Find a smooth solution u under constraint

$$\int |u - z|^2 = \sigma^2$$

- If the solution is to minimize H1 norm $\int |\nabla u|^2$ we call it H1 regularization

Variational approach to denoising-2

- H1 denoising $$\min_u \int |u-z|^2 + \alpha \int |\nabla u|^2$$

Regularization penalty

Euler-Lagrange equation

$$\alpha \Delta u - (u-z) = 0$$

- Total variation denosing $$\min_u \int |u-z|^2 + \alpha \int |\nabla u|$$

Euler-Lagrange equation

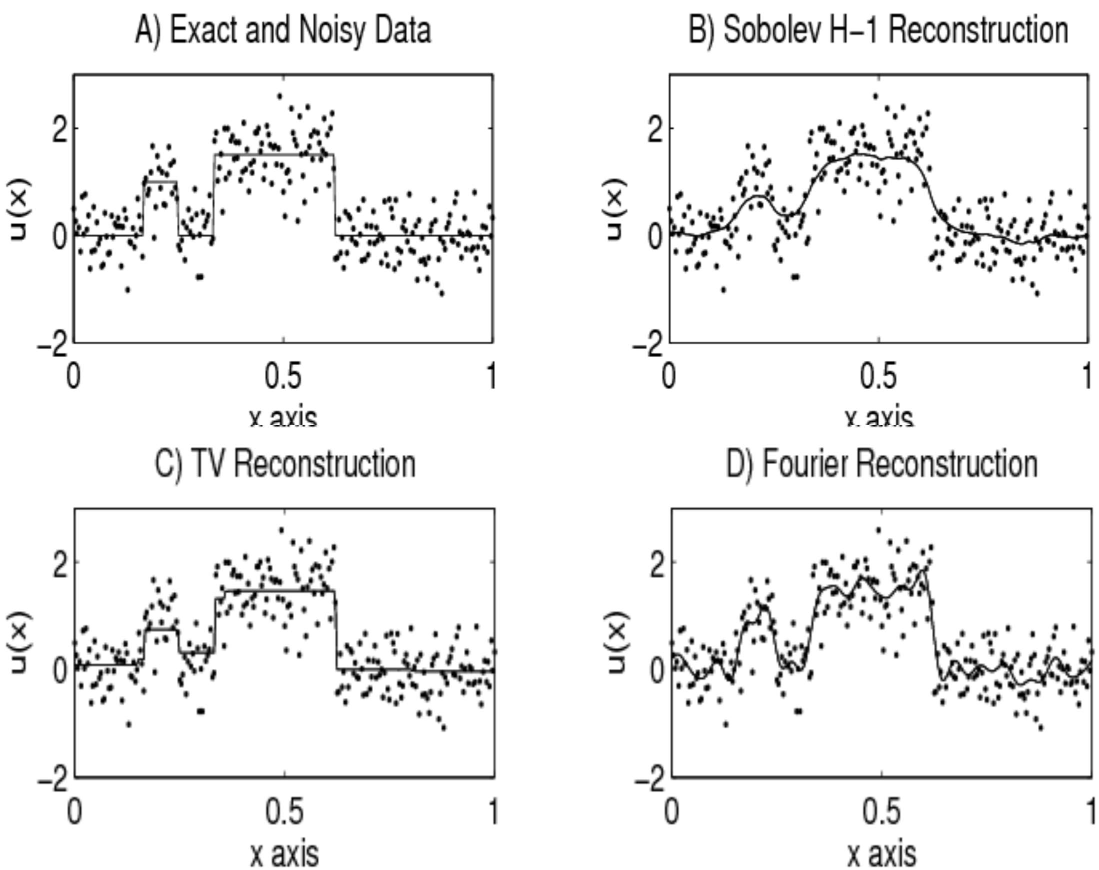
$$\alpha \nabla \cdot \left(\frac{\nabla u}{|\nabla u|} \right) - (u-z) = 0$$

Why Total variation denoising

- TV norm: Keep edge sharp

Rudin, Osher, Fatemi

TV norm is insensitive to jumps (edges)



Picture by Vogel and Oman

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Blur model-1

- Convolution

$$g(x) = [h * f](x) := \int h(x-y) f(y) dy$$

- If h is a positive weight, then h*f is an averaging process, i.e. blurring

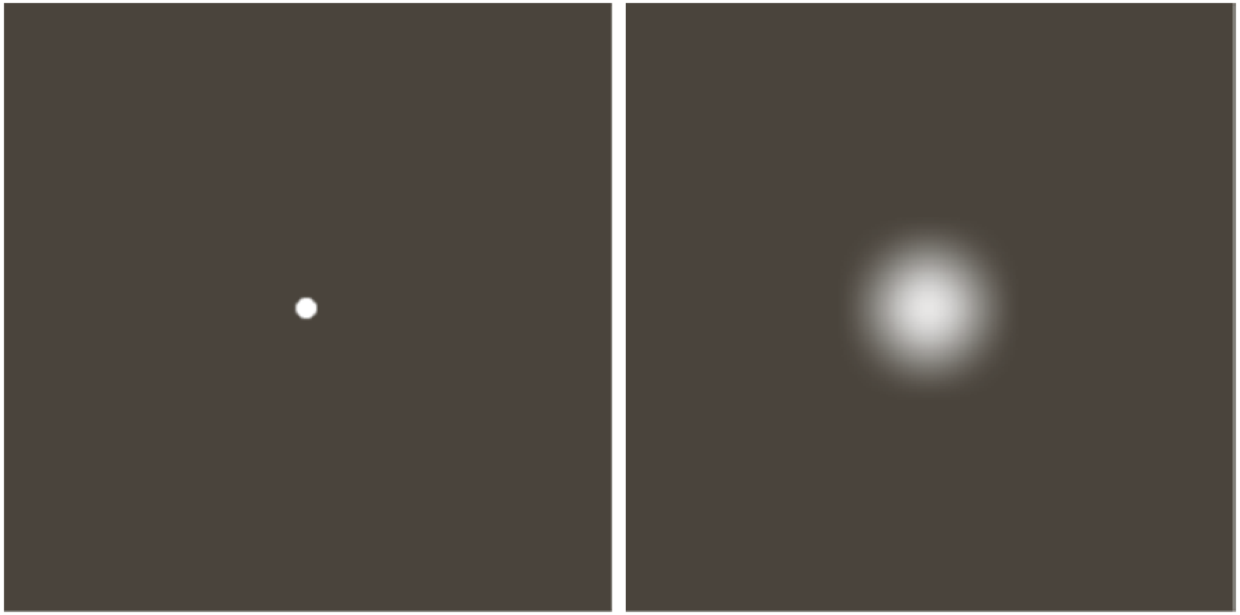
- Example: Finite size mask $h_{s,t} = \frac{1}{16}$

1	2	1
2	4	2
1	2	1

Blur model-2

a b

FIGURE 5.24 Degradation estimation by impulse characterization. (a) An impulse of light (shown magnified). (b) Imaged (degraded) impulse.



$$g = h * f$$

$$\hat{g} = \hat{h} \cdot \hat{f}$$

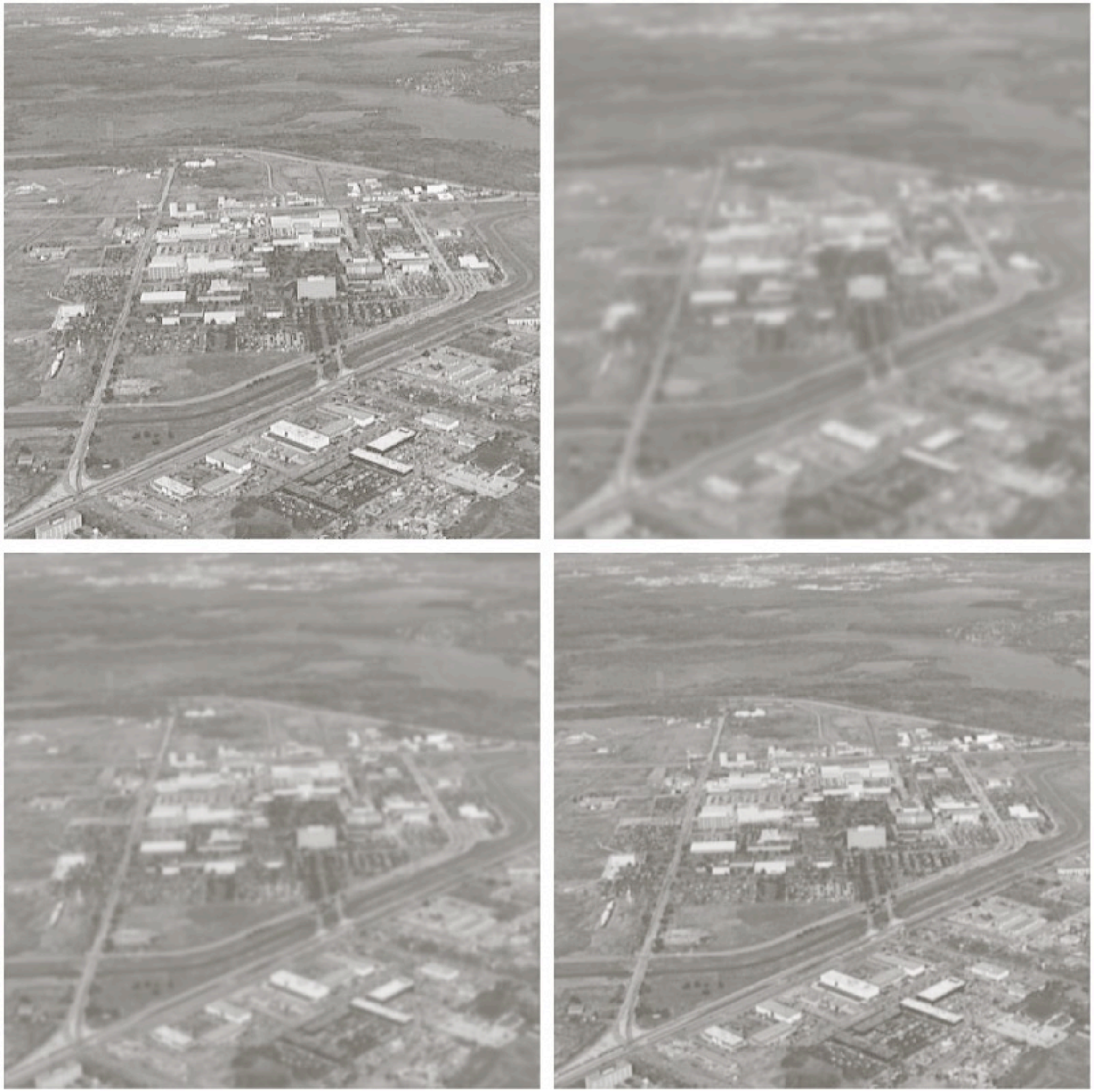
Atmospheric turbulence $\qquad \hat{h}(\xi,\eta) = e^{-k(\xi^2+\eta^2)^{5/6}}$

Gaussian model $\qquad \hat{h}(\xi,\eta) = e^{-k(\xi^2+\eta^2)}$

Blur model-3

a b
c d

FIGURE 5.25 Illustration of the atmospheric turbulence model. (a) Negligible turbulence. (b) Severe turbulence, $k = 0.0025$. (c) Mild turbulence, $k = 0.001$. (d) Low turbulence, $k = 0.00025$. (Original image courtesy of NASA.)



Blur model -4

$$g(x,y)=\int_0^T f(x-x_0(t),y-y_0(t))dt$$

K : Translation

Blur model-5

$$g = h * f + n$$

h: Blur operator
n: noise
f: true image

Deblur methods

- Deconvolution in frequency domain
  - Inverse filtering
  - Wiener filtering
  - …
- Deconvolution via wavelets
- Variational approach

Deconvolution

$$\tilde{f} = k * g$$

- Inverse filtering

$$g = h * f + n \Rightarrow \hat{g} = \hat{h} \cdot \hat{f} + \hat{n}$$

$$\hat{\tilde{f}} = \hat{k}\hat{g} = \frac{1}{\hat{h}}\hat{g}$$

- Wiener filtering

$$\hat{k} = \frac{\overline{\hat{h}} E\{\hat{f}, \hat{f}\}}{|\hat{h}|^2 \, E\{\hat{f}, \hat{f}\} + E\{\hat{n}, \hat{n}\}}$$

Deblur-1

a b c

FIGURE 5.28 Comparison of inverse and Wiener filtering. (a) Result of full inverse filtering of Fig. 5.25(b). (b) Radially limited inverse filter result. (c) Wiener filter result.

Wiener filter

Deblur-2

Blur+noise | Inverse filter | Wiener filter

Deblur methods

- Deconvolution in frequency domain
  - Inverse filtering
  - Wiener filtering
  - …
- Deconvolution via wavelets

- Variational approach

Debur via TV regularization-1

- Blur model $g = h * f + n$

- Total variation regularization:

$$\min_f \alpha \int |\nabla f| + \int |h * f - g|^2$$

- Alternative formulation

$$\min_{f,w} \alpha \int |w| + \beta \int |\nabla f - w|^2 + \int |h * f - g|^2$$

Y Wang et al.

Deblur via TV regularization-2

Y Wang et al.

Deblur via TV regularization-3

Y Wang et al

Imaging Sciences

- Image Acquistion (Imaging)
  - human vision, Optics, Radar imaging, Ultrasound, MRI, X-ray CT,…
- Image Processing

$$I_{input} \xrightarrow{T} I_{output} = T[I_{input}]$$

- Image Interpretation (Visual Intelligence)

What is imaging?

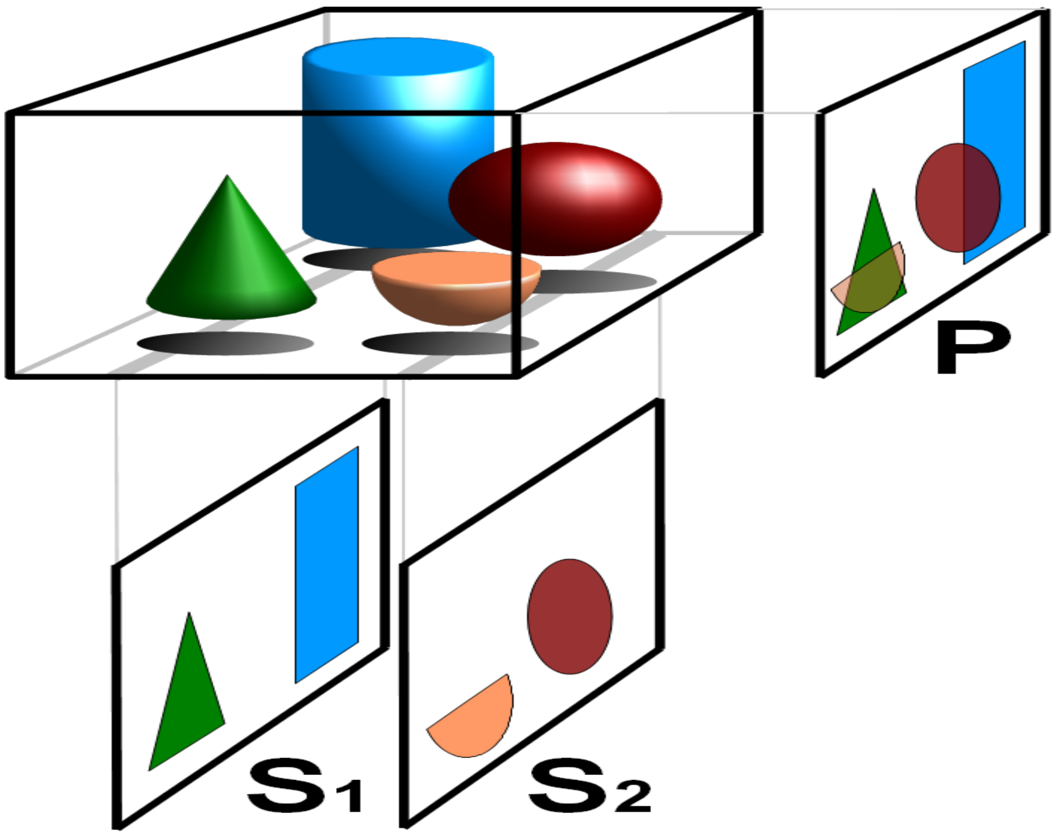
- Use physical methods to get geometrical or physical properties of the objects
  - Geometry: shape, morphology, structure,….
  - Physical properties:
    - Mechanical: density, pressure, velocity, concentration, viscosity, diffusion coefficients,…
    - Electrical: potential, current, impedance, conductivity, resistance,
    - Optical: absorption/reflection…
    - nuclear

Medical imaging (Wiki)

- 1 Projection radiography
- 2 Tomography
- 3 Ultrasound
- 4 Fluoroscopy
- 5 Magnetic resonance imaging (MRI)
- 6 Nuclear medicine
- 7 Positron emission tomography (PET)
- 8 Photoacoustic imaging

Projection radiography

Tomography



Basic principle of tomography: superposition free tomographic cross sections S1 and S2 compared with the projected image P

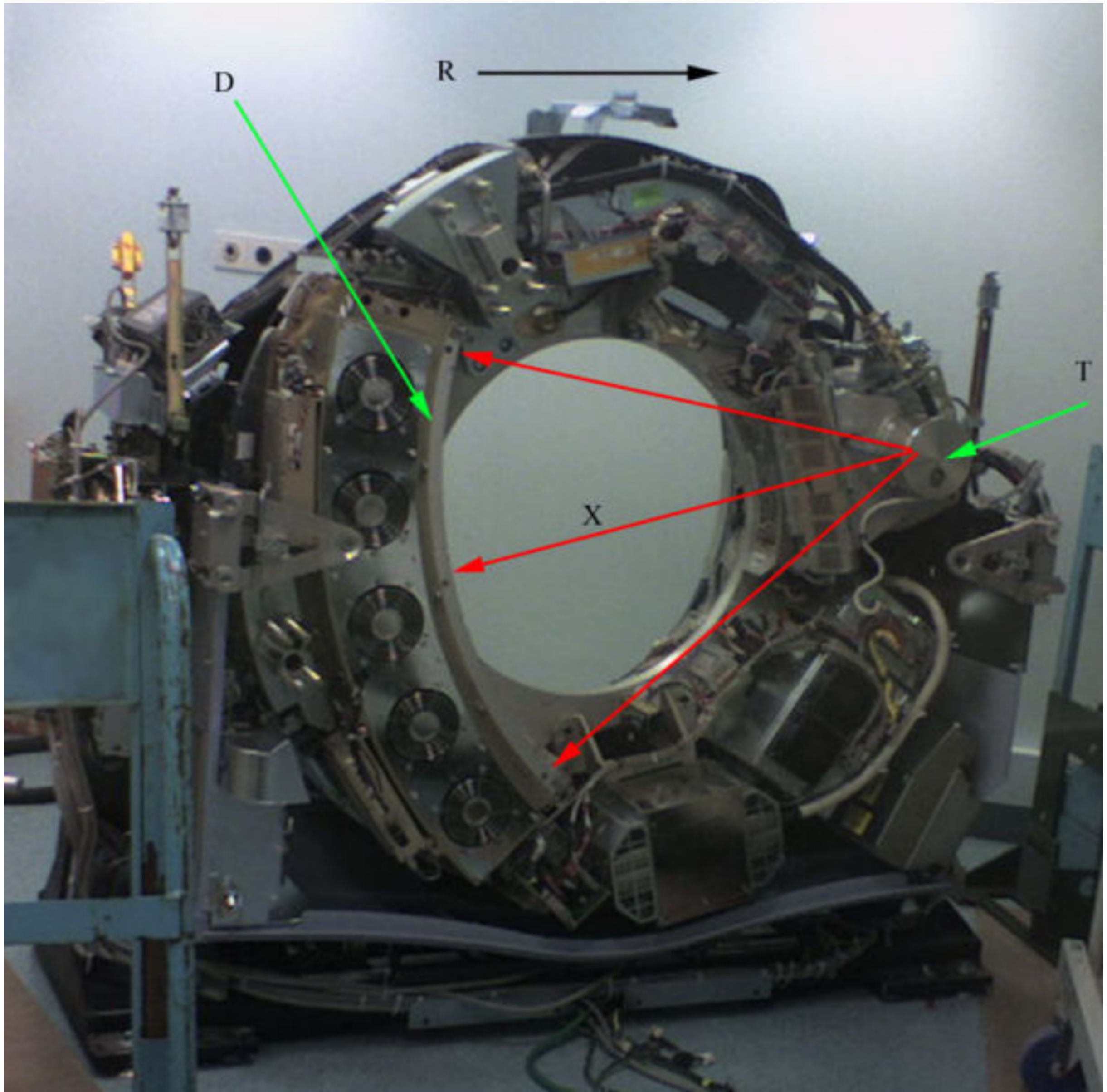
Type of Tomography-1

- Atom probe tomography (APT)
- Computed tomography (CT)
- Confocal laser scanning microscopy (LSCM)
- Cryo-electron tomography (Cryo-ET)
- Electrical capacitance tomography (ECT)
- Electrical resistivity tomography (ERT)
- Electrical impedance tomography (EIT)
- Functional magnetic resonance imaging (fMRI)
- Magnetic induction tomography (MIT)
- Magnetic resonance imaging (MRI), formerly known as *magnetic resonance tomography (MRT) or nuclear magnetic resonance tomography*

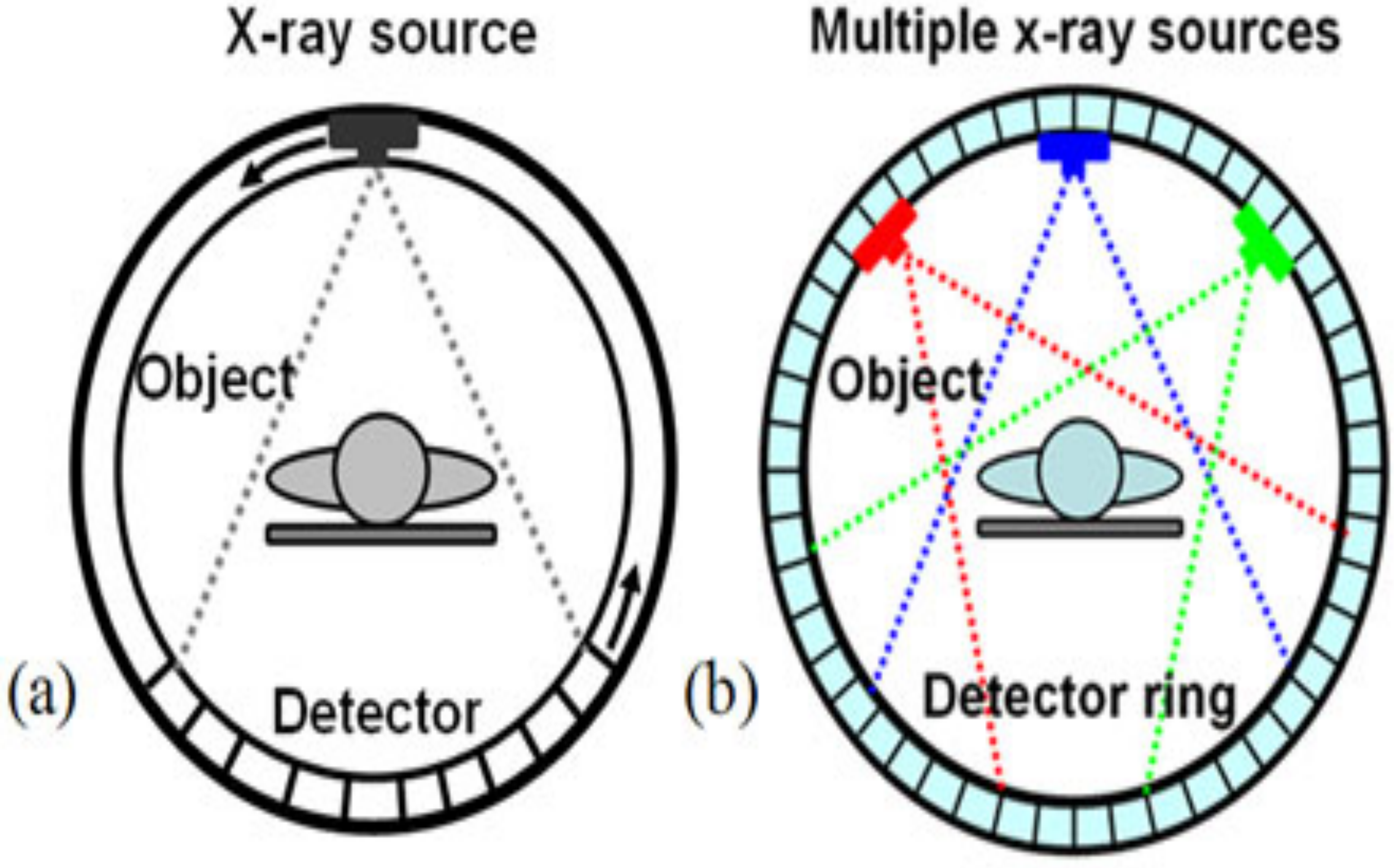
Type of Tomography-2

- Optical coherence tomography (OCT)
- Process tomography (PT)
- Positron emission tomography (PET)
- Positron emission tomography - computed tomography (PET-CT)
- Quantum tomography
- Single photon emission computed tomography (SPECT)
- Seismic tomography
- X-ray tomography (CT, CATScan)
- Photoacoustic tomography (PAT), also known as Optoacoustic Tomography (OAT) or Thermoacoustic Tomography (TAT)
- Zeeman-Doppler imaging

X-ray Computed Tomograph



X-ray CT


X-ray source
Object
(a)
Detector
Multiple x-ray sources
Object
(b)
Detector ring

Nobel winners for CT (1979)

Godfrey Hounsfield

Allan McLeod Cormack

Image Reconstruction

- Tomographic reconstruction :
  - Radon transform

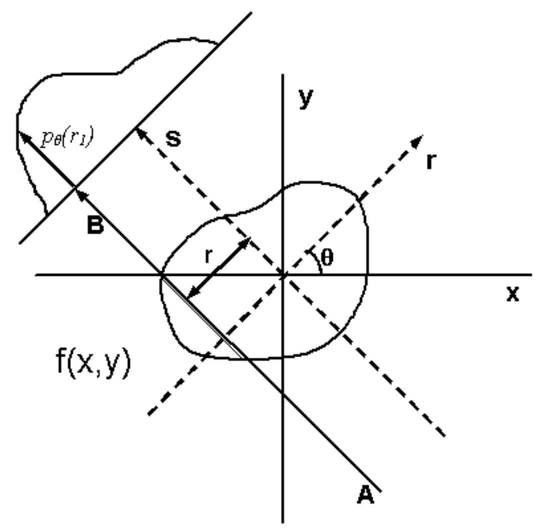
$$Rf(\theta, r) = \int_{x\cdot\theta=r} f(x)dx, \;\; \theta \in S^1$$

  - Imaging model

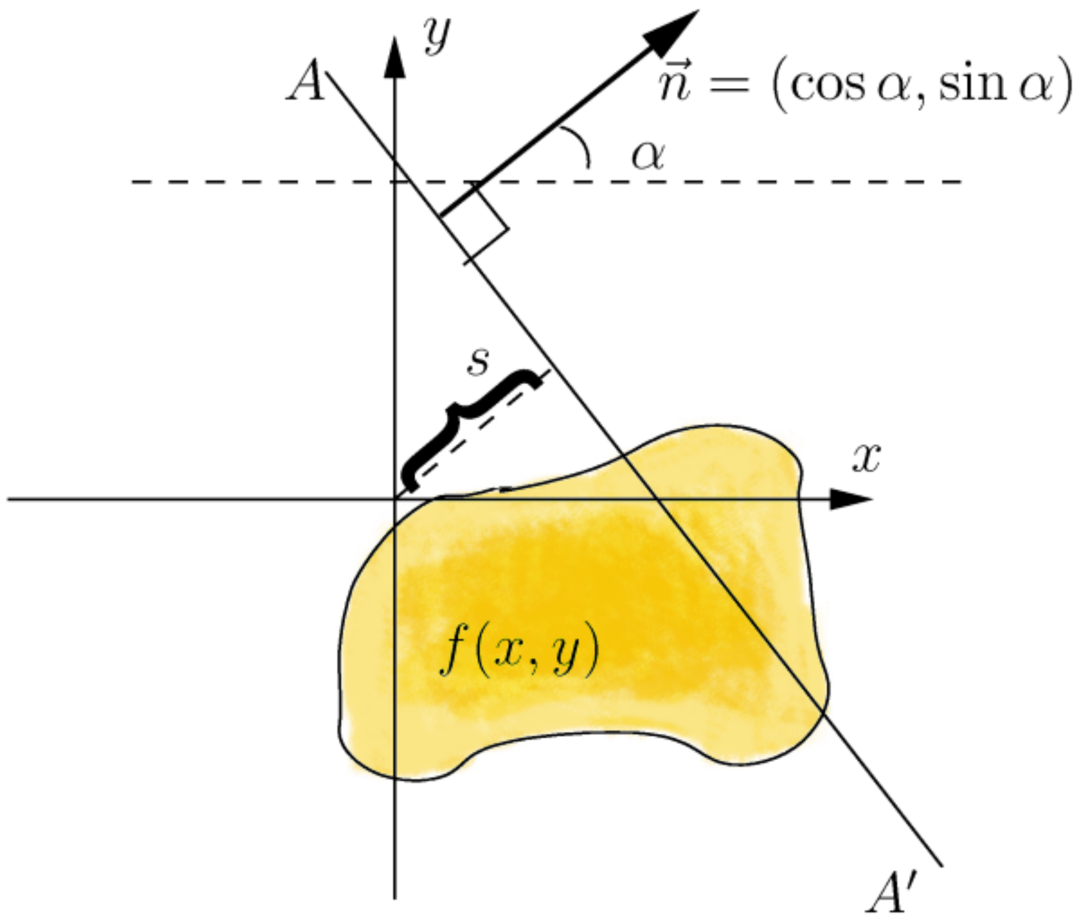
$$z = Ru + n$$

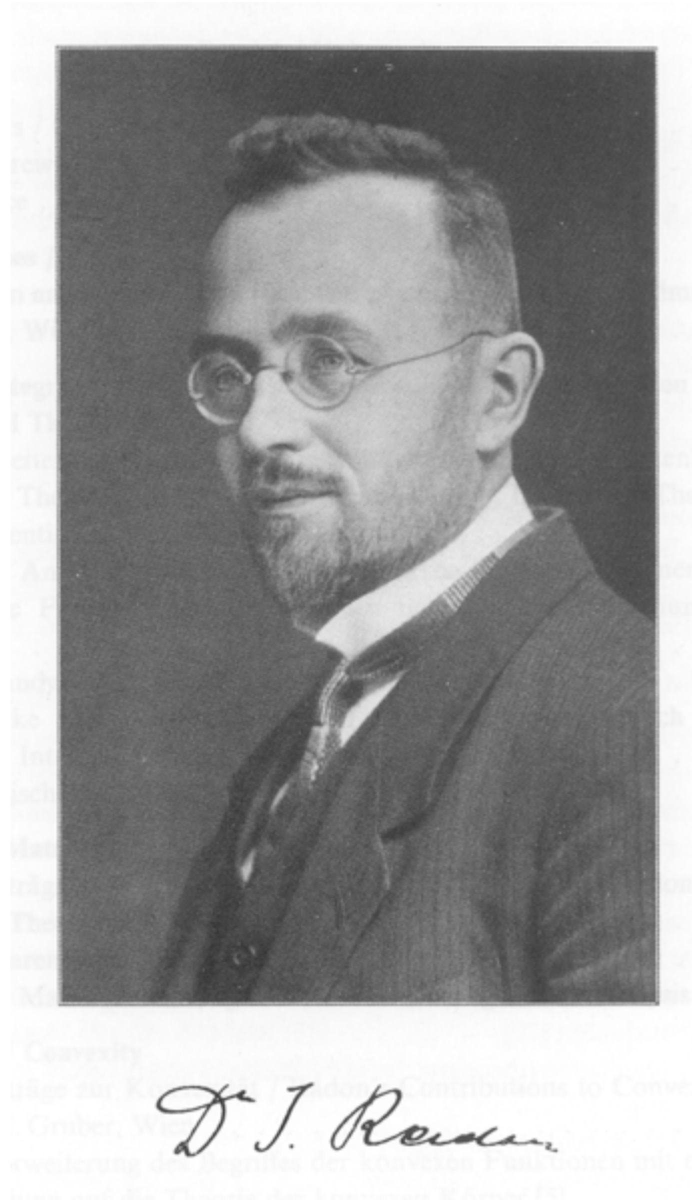
  - Image reconstruction

$$\text{Given } z, \text{ reconstruct } u$$

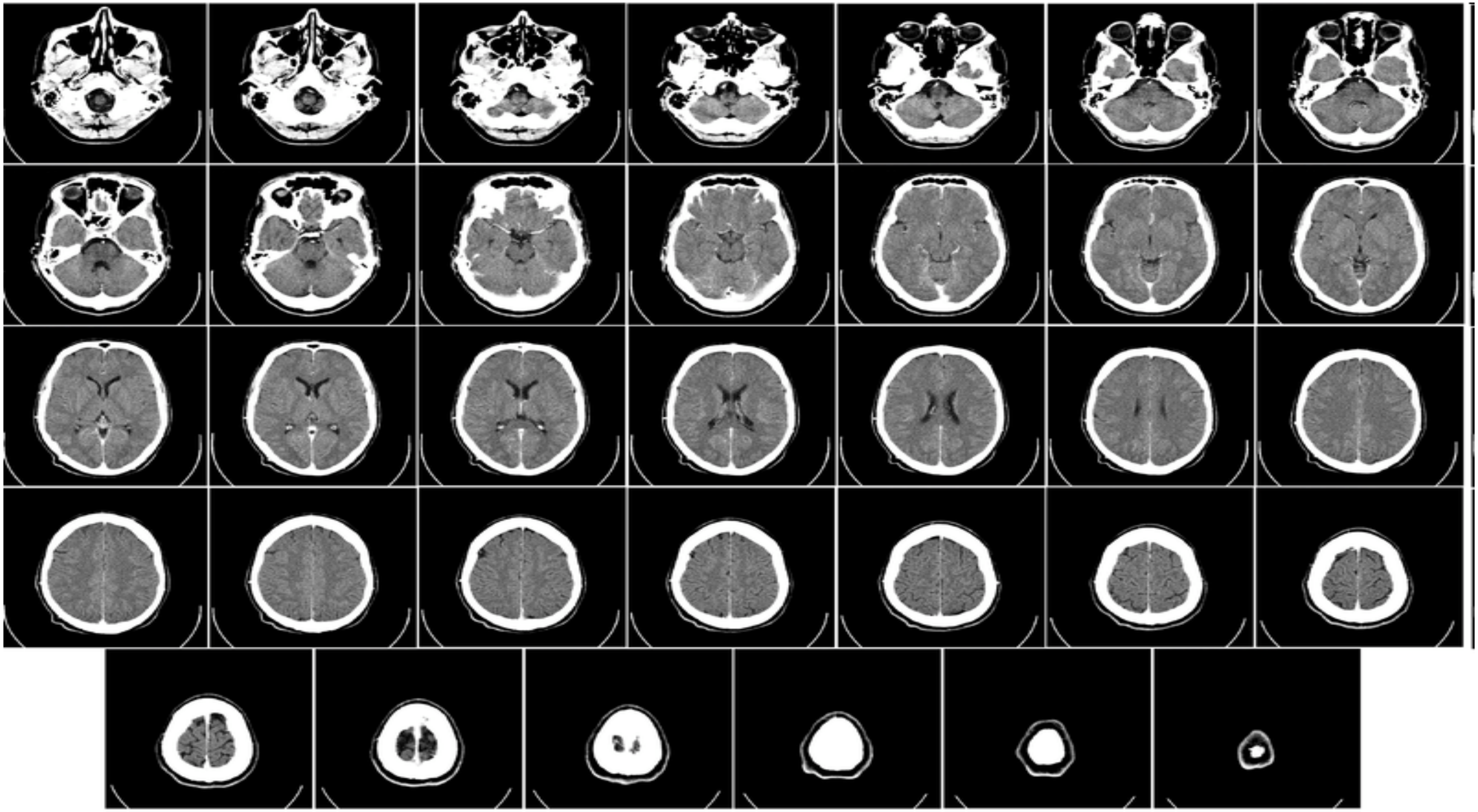


Radon transform

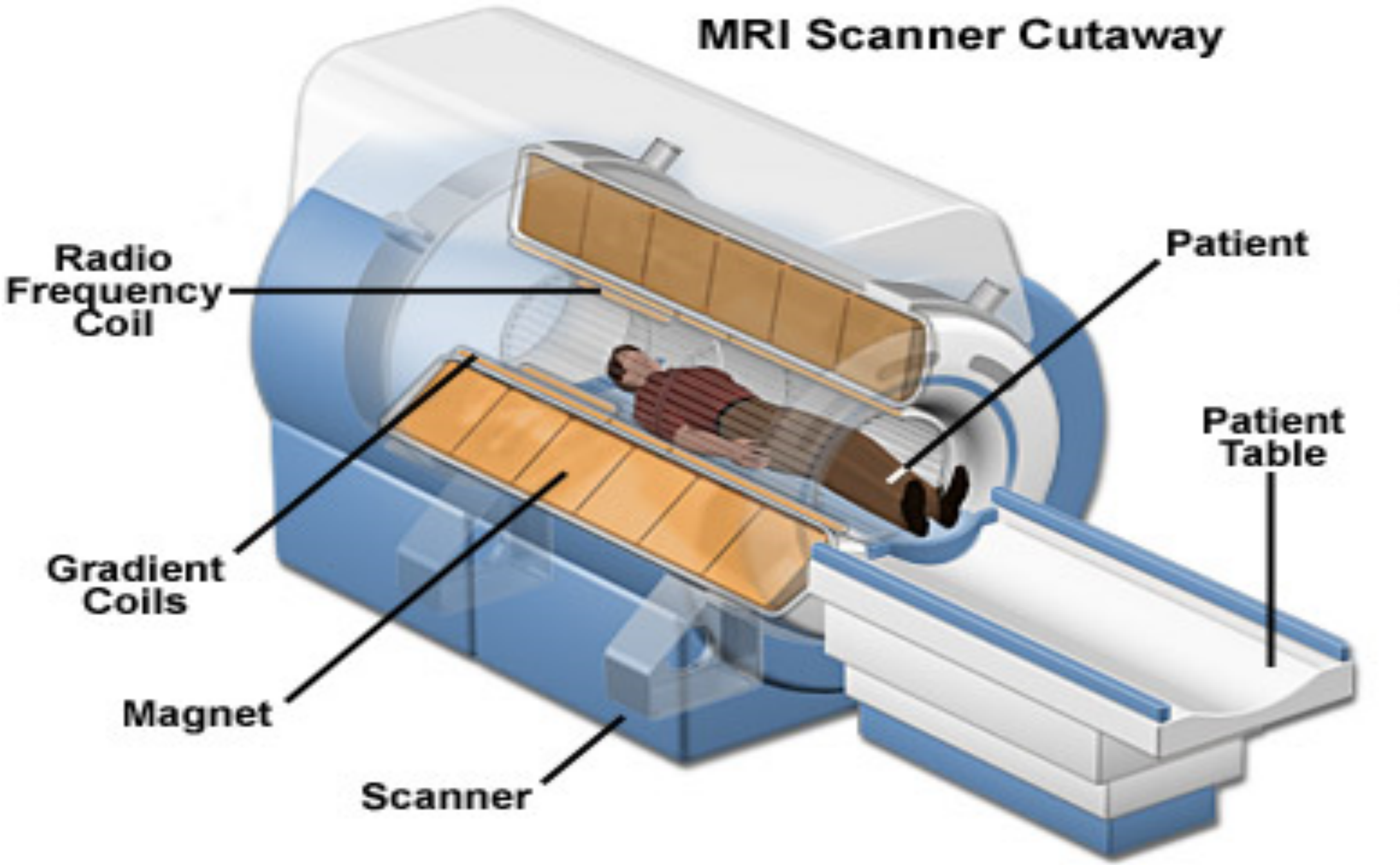




Reconstructed images by CT



Magnetic Resonance Imaging (MRI)



MRI history

The Nobel Prize in Physics 1952

"for their development of new methods for nuclear magnetic precision measurements and discoveries in connection therewith"

Felix Bloch

1/2 of the prize

USA

Stanford University
Stanford, CA, USA

Edward Mills Purcell

1/2 of the prize

USA

Harvard University
Cambridge, MA, USA

The Nobel Prize in Physiology or Medicine 2003

"for their discoveries concerning magnetic resonance imaging"

Paul C. Lauterbur

1/2 of the prize

USA

University of Illinois
Urbana, IL, USA

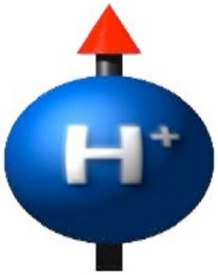
Sir Peter Mansfield

1/2 of the prize

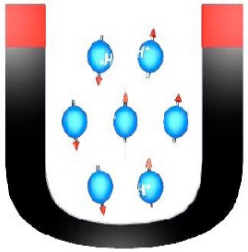
United Kingdom

University of Nottingham, School of Physics and Astronomy
Nottingham, United Kingdom

Basic Principles of Nuclear Magnetic Resonance



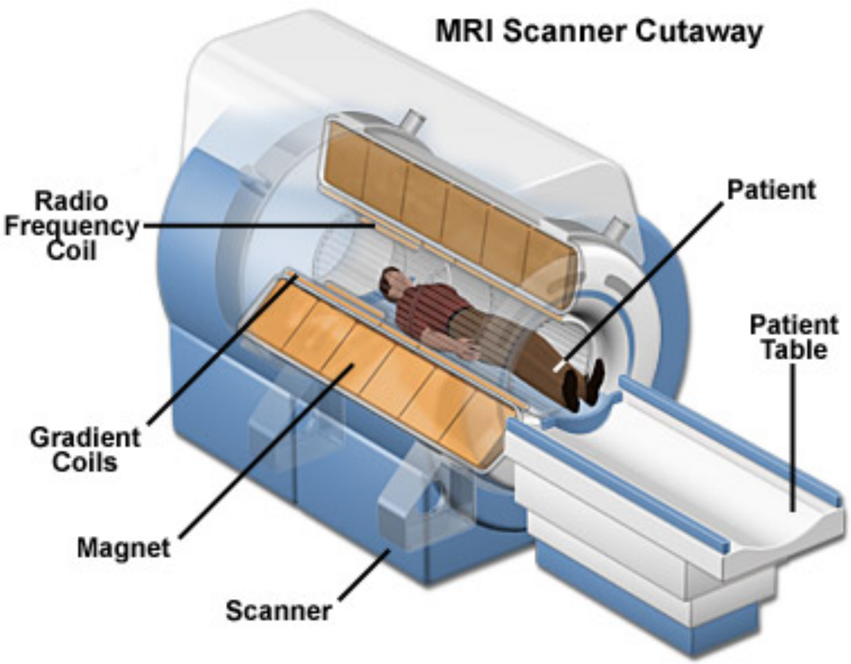
- Atoms with odd number of protons and/or neutrons possess nuclear spin angular momentum S
- Associated with S is a magnetic dipole moment
- Magnetic dipole moment rotates under external magnetic field, exhibit magnetic resonance phenomena

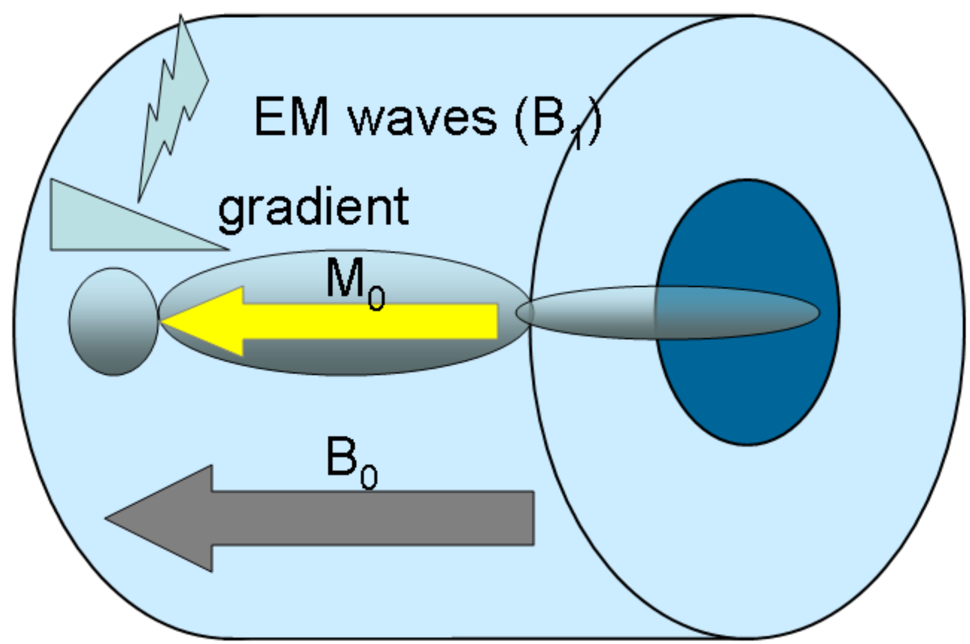


- The variation of rotation of spins generates magnetic fluxes and can be recorded
- Hydrogen H+ atoms are abundant in biological specimens

MRI:
use magnetic fields to perform

- Relaxation: Main field B0
- Excitation: Radio Frequency (RF) field B1
- Fourier transform: Gradient field G

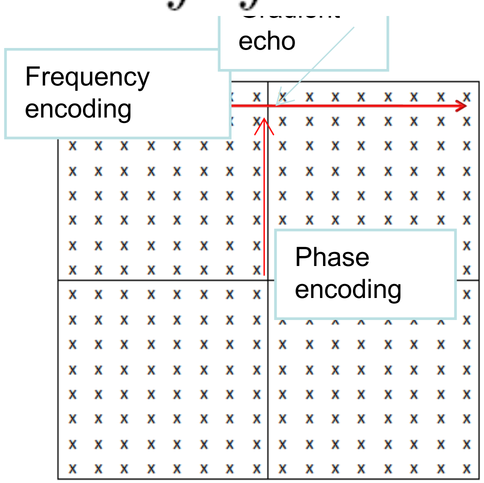


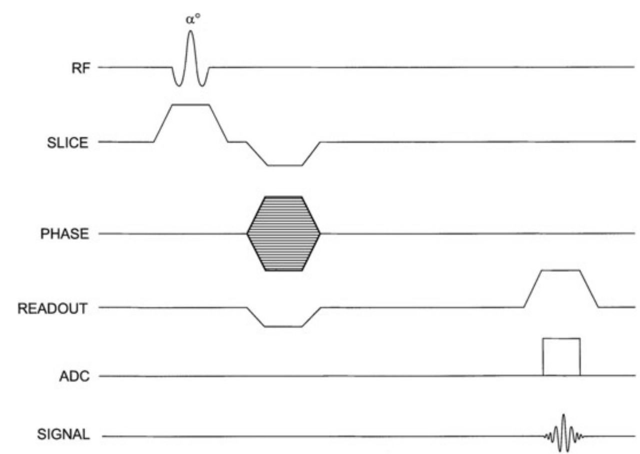


MRI is a Fourier integrator

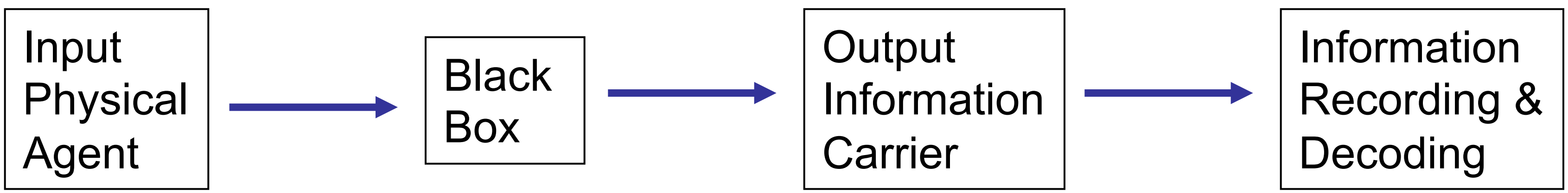
- RF excitation selects a slice of magnetic dipoles
- The gradient field generates Fourier transform of

$$\begin{aligned} s(t) &= \int\!\!\int m(x,y) \exp\left(-i\gamma \int_0^t \mathbf{G}(\tau)\cdot \mathbf{r}\, d\tau\right) dx\, dy \\ &= \int\!\!\int m(x,y) e^{-2i\pi[k_x(t)x + k_y(t)y]}\, dx\, dy \end{aligned}$$





Magnetic Resonance Imaging



EM waves

Pulse sequences

Transmit coils

Contrast agents

Magnetic dipoles of mobile protons

EM waves

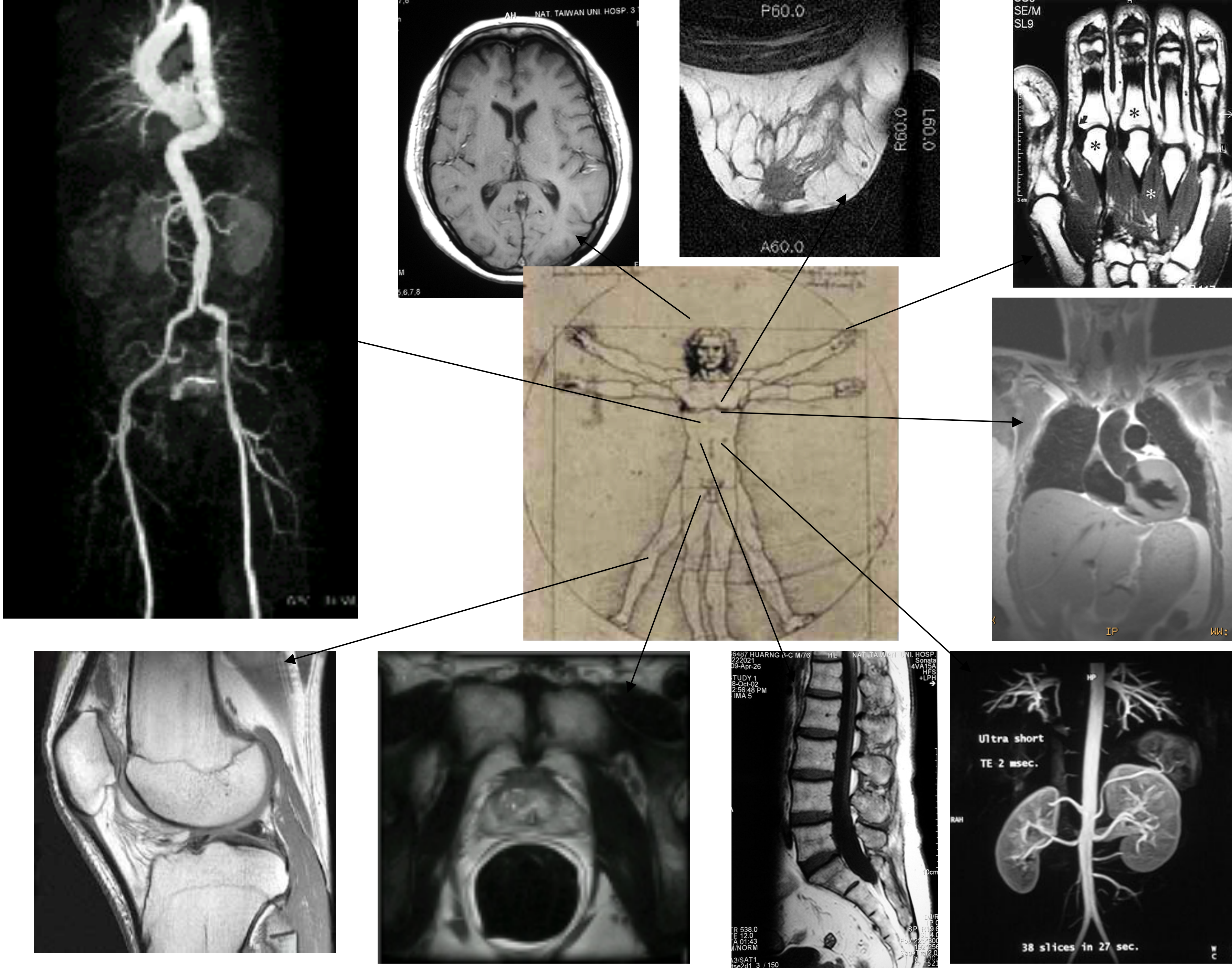
- T1 & T2
- Flow
- Diffusion
- Perfusion
- Temperature
- Cell tracking
- Molecules

Receive coils

Image reconstruction

Data processing

Data analysis


NAT. TAIWAN UNI. HOSP.
P60.0
A60.0
R60.0
L60.0
SE/M
SL9
IP
Ultra short
TE 2 msec.
38 slices in 27 sec.

Summary of Imaging Sciences

- Imaging (data acquisition): CT, MRI
  - Solving inverse problems
- Image processing:
  - Enhancement (contrast enhancement, denoising, deblurring,…)
  - Segmentation (edge detection, active contours,…)
- Image analysis, image interpretation

Image science and mathematics

- Image science is important in medicine
- Low dose, high resolution imaging methods are needed
- Image science needs mathematics

- Thank you for your attention.