2. $\int_0^5 ye^{-0.6y}\,dy \quad \begin{bmatrix} u = y, & dv = e^{-0.6y}dy, \\ du = dy & v = -\frac{5}{3}e^{-0.6y} \end{bmatrix} = \left[-\frac{5}{3}ye^{-0.6y}\right]_0^5 - \int_0^5 \left(-\frac{5}{3}e^{-0.6y}\right)dy = -\frac{25}{3}e^{-3} - \frac{25}{9}\left[e^{-0.6y}\right]_0^5$

$= -\frac{25}{3}e^{-3} - \frac{25}{9}(e^{-3} - 1) = -\frac{25}{3}e^{-3} - \frac{25}{9}e^{-3} + \frac{25}{9} = \frac{25}{9} - \frac{100}{9}e^{-3}$

6. $\dfrac{1}{y^2 - 4y - 12} = \dfrac{1}{(y-6)(y+2)} = \dfrac{A}{y-6} + \dfrac{B}{y+2} \quad \Rightarrow \quad 1 = A(y+2) + B(y-6)$. Letting $y = -2 \quad \Rightarrow \quad B = -\frac{1}{8}$ and letting $y = 6 \quad \Rightarrow \quad A = \frac{1}{8}$. So $\displaystyle\int \frac{1}{y^2 - 4y - 12}\,dy = \int \left(\frac{1/8}{y-6} + \frac{-1/8}{y+2}\right)dy = \frac{1}{8}\ln|y-6| - \frac{1}{8}\ln|y+2| + C.$

7. Let $u = \ln t$, $du = dt/t$. Then $\displaystyle\int \frac{\sin(\ln t)}{t}\,dt = \int \sin u\,du = -\cos u + C = -\cos(\ln t) + C.$

8. Let $u = \sqrt{e^x - 1}$, so that $u^2 = e^x - 1$, $2u\,du = e^x\,dx$, and $e^x = u^2 + 1$. Then

$$\int \frac{1}{\sqrt{e^x - 1}}\,dx = \int \frac{1}{u}\frac{2u\,du}{u^2+1} = 2\int \frac{1}{u^2+1}\,du = 2\tan^{-1}u + C = 2\tan^{-1}\sqrt{e^x - 1} + C.$$

17. Integrate by parts with $u = x$, $dv = \sec x\,\tan x\,dx \quad \Rightarrow \quad du = dx$, $v = \sec x$:

$\int x\sec x\,\tan x\,dx = x\sec x - \int \sec x\,dx \overset{14}{=} x\sec x - \ln|\sec x + \tan x| + C.$

22. Let $x = \sqrt{t}$, so that $x^2 = t$ and $2x\,dx = dt$. Then

$$\begin{aligned}
\int te^{\sqrt{t}}\,dt &= \int x^2e^x\,(2x\,dx) = \int 2x^3e^x\,dx & &\begin{bmatrix} u_1 = 2x^3, & dv_1 = e^x\,dx, \\ du_1 = 6x^2\,dx & v_1 = e^x \end{bmatrix} \\
&= 2x^3e^x - \int 6x^2e^x\,dx & &\begin{bmatrix} u_2 = 6x^2, & dv_2 = e^x\,dx, \\ du_2 = 12x\,dx & v_2 = e^x \end{bmatrix} \\
&= 2x^3e^x - \left(6x^2e^x - \int 12xe^x\,dx\right) & &\begin{bmatrix} u_3 = 12x, & dv_3 = e^x\,dx, \\ du_3 = 12\,dx & v_3 = e^x \end{bmatrix} \\
&= 2x^3e^x - 6x^2e^x + \left(12xe^x - \int 12e^x\,dx\right) = 2x^3e^x - 6x^2e^x + 12xe^x - 12e^x + C \\
&= 2e^x(x^3 - 3x^2 + 6x - 6) + C = 2e^{\sqrt{t}}\left(t\sqrt{t} - 3t + 6\sqrt{t} - 6\right) + C
\end{aligned}$$

27. $\int_0^{\pi/2} \cos^3 x\,\sin 2x\,dx = \int_0^{\pi/2} \cos^3 x\,(2\sin x\,\cos x)\,dx = \int_0^{\pi/2} 2\cos^4 x\,\sin x\,dx = \left[-\frac{2}{5}\cos^5 x\right]_0^{\pi/2} = \frac{2}{5}$

28. Let $u = \sqrt[3]{x}$. Then $x = u^3$, $dx = 3u^2du \quad \Rightarrow$

$$\int \frac{\sqrt[3]{x}+1}{\sqrt[3]{x}-1}\,dx = \int \frac{u+1}{u-1}3u^2du = 3\int \left(u^2 + 2u + 2 + \frac{2}{u-1}\right)du$$

$$= u^3 + 3u^2 + 6u + 6\ln|u-1| + C = x + 3x^{2/3} + 6\sqrt[3]{x} + 6\ln|\sqrt[3]{x} - 1| + C$$

42. $\displaystyle\int_1^\infty \frac{\ln x}{x^4}\,dx = \lim_{t\to\infty}\int_1^t \frac{\ln x}{x^4}\,dx \quad \begin{bmatrix} u = \ln x, & dv = dx/x^4, \\ du = dx/x & v = -1/(3x^3) \end{bmatrix}$

$$= \lim_{t\to\infty}\left[-\frac{\ln x}{3x^3}\right]_1^t + \int_1^t \frac{1}{3x^4}\,dx = \lim_{t\to\infty}\left(-\frac{\ln t}{3t^3} + 0 + \left[\frac{-1}{9x^3}\right]_1^t\right) \overset{\text{H}}{=} \lim_{t\to\infty}\left(-\frac{1}{9t^3} + \left[\frac{-1}{9t^3} + \frac{1}{9}\right]\right)$$

$$= 0 + 0 + \frac{1}{9} = \frac{1}{9}$$

45. $\displaystyle\int_0^4 \frac{\ln x}{\sqrt{x}}\,dx = \lim_{t\to 0^+}\int_t^4 \frac{\ln x}{\sqrt{x}}\,dx \overset{*}{=} \lim_{t\to 0^+}\Big[2\sqrt{x}\,\ln x - 4\sqrt{x}\Big]_t^4$

$\displaystyle = \lim_{t\to 0^+}\left[(2\cdot 2\ln 4 - 4\cdot 2) - \left(2\sqrt{t}\,\ln t - 4\sqrt{t}\right)\right] \overset{**}{=} (4\ln 4 - 8) - (0-0) = 4\ln 4 - 8$

(∗) Let $u = \ln x$, $dv = \dfrac{1}{\sqrt{x}}\,dx \quad\Rightarrow\quad du = \dfrac{1}{x}\,dx$, $v = 2\sqrt{x}$. Then

$$\int \frac{\ln x}{\sqrt{x}}\,dx = 2\sqrt{x}\,\ln x - 2\int \frac{dx}{\sqrt{x}} = 2\sqrt{x}\,\ln x - 4\sqrt{x} + C$$

(∗∗) $\displaystyle\lim_{t\to 0^+}\left(2\sqrt{t}\,\ln t\right) = \lim_{t\to 0^+}\frac{2\ln t}{t^{-1/2}} \overset{\mathrm{H}}{=} \lim_{t\to 0^+}\frac{2/t}{-\frac{1}{2}t^{-3/2}} = \lim_{t\to 0^+}\left(-4\sqrt{t}\right) = 0$

63. $f(x) = \dfrac{1}{\ln x}$, $\Delta x = \dfrac{b-a}{n} = \dfrac{4-2}{10} = \dfrac{1}{5}$

(a) $T_{10} = \frac{1}{5\cdot 2}\{f(2) + 2[f(2.2) + f(2.4) + \cdots + f(3.8)] + f(4)\} \approx 1.925444$

(b) $M_{10} = \frac{1}{5}[f(2.1) + f(2.3) + f(2.5) + \cdots + f(3.9)] \approx 1.920915$

(c) $S_{10} = \frac{1}{5\cdot 3}[f(2) + 4f(2.2) + 2f(2.4) + \cdots + 2f(3.6) + 4f(3.8) + f(4)] \approx 1.922470$

71. $\dfrac{x^3}{x^5+2} \le \dfrac{x^3}{x^5} = \dfrac{1}{x^2}$ for x in $[1,\infty)$. $\displaystyle\int_1^\infty \frac{1}{x^2}\,dx$ is convergent by (7.8.2) with $p = 2 > 1$. Therefore, $\displaystyle\int_1^\infty \frac{x^3}{x^5+2}\,dx$ is convergent by the Comparison Theorem.