9. f is a rational function, so it is continuous on its domain. Since f is defined at $(1, 1)$, we use direct substitution to evaluate the limit: $\lim\limits_{(x,y)\to(1,1)} \dfrac{2xy}{x^2+2y^2} = \dfrac{2(1)(1)}{1^2+2(1)^2} = \dfrac{2}{3}$.

14. $u = e^{-r}\sin 2\theta \quad\Rightarrow\quad u_r = -e^{-r}\sin 2\theta,\ u_\theta = 2e^{-r}\cos 2\theta$

22. $v = r\cos(s+2t) \quad\Rightarrow\quad v_r = \cos(s+2t),\ v_s = -r\sin(s+2t),\ v_t = -2r\sin(s+2t),\ v_{rr} = 0,\ v_{ss} = -r\cos(s+2t),$ $v_{tt} = -4r\cos(s+2t),\ v_{rs} = v_{sr} = -\sin(s+2t),\ v_{rt} = v_{tr} = -2\sin(s+2t),\ v_{st} = v_{ts} = -2r\cos(s+2t)$

26. (a) $z_x = e^x\cos y \quad\Rightarrow\quad z_x(0,0) = 1$ and $z_y = -e^x\sin y \quad\Rightarrow\quad z_y(0,0) = 0$, so an equation of the tangent plane is $z - 1 = 1(x-0) + 0(y-0)$ or $z = x+1$.

(b) A normal vector to the tangent plane (and the surface) at $(0, 0, 1)$ is $\langle 1, 0, -1\rangle$. Then parametric equations for the normal line there are $x = t$, $y = 0$, $z = 1 - t$, and symmetric equations are $x = 1 - z$, $y = 0$.

33. $f(x,y,z) = x^3\sqrt{y^2+z^2} \quad\Rightarrow\quad f_x(x,y,z) = 3x^2\sqrt{y^2+z^2},\ f_y(x,y,z) = \dfrac{yx^3}{\sqrt{y^2+z^2}},\ f_z(x,y,z) = \dfrac{zx^3}{\sqrt{y^2+z^2}},$

so $f(2,3,4) = 8(5) = 40$, $f_x(2,3,4) = 3(4)\sqrt{25} = 60$, $f_y(2,3,4) = \frac{3(8)}{\sqrt{25}} = \frac{24}{5}$, and $f_z(2,3,4) = \frac{4(8)}{\sqrt{25}} = \frac{32}{5}$. Then the linear approximation of f at $(2, 3, 4)$ is

$$\begin{aligned} f(x,y,z) &\approx f(2,3,4) + f_x(2,3,4)(x-2) + f_y(2,3,4)(y-3) + f_z(2,3,4)(z-4) \\ &= 40 + 60(x-2) + \tfrac{24}{5}(y-3) + \tfrac{32}{5}(z-4) = 60x + \tfrac{24}{5}y + \tfrac{32}{5}z - 120 \end{aligned}$$

Then $(1.98)^3\sqrt{(3.01)^2+(3.97)^2} = f(1.98, 3.01, 3.97) \approx 60(1.98) + \frac{24}{5}(3.01) + \frac{32}{5}(3.97) - 120 = 38.656$.

35. $\dfrac{du}{dp} = \dfrac{\partial u}{\partial x}\dfrac{dx}{dp} + \dfrac{\partial u}{\partial y}\dfrac{dy}{dp} + \dfrac{\partial u}{\partial z}\dfrac{dz}{dp} = 2xy^3(1+6p) + 3x^2y^2(pe^p + e^p) + 4z^3(p\cos p + \sin p)$

36. $\dfrac{\partial v}{\partial s} = \dfrac{\partial v}{\partial x}\dfrac{\partial x}{\partial s} + \dfrac{\partial v}{\partial y}\dfrac{\partial y}{\partial s} = \left(2x\sin y + y^2e^{xy}\right)(1) + \left(x^2\cos y + xye^{xy} + e^{xy}\right)(t).$

$s = 0, t = 1 \quad\Rightarrow\quad x = 2, y = 0$, so $\dfrac{\partial v}{\partial s} = 0 + (4+1)(1) = 5.$

$\dfrac{\partial v}{\partial t} = \dfrac{\partial v}{\partial x}\dfrac{\partial x}{\partial t} + \dfrac{\partial v}{\partial y}\dfrac{\partial y}{\partial t} = \left(2x\sin y + y^2e^{xy}\right)(2) + \left(x^2\cos y + xye^{xy} + e^{xy}\right)(s) = 0 + 0 = 0.$

42. $F(x,y,z) = e^{xyz} - yz^4 - x^2z^3 = 0$, so $\dfrac{\partial z}{\partial x} = -\dfrac{F_x}{F_z} = -\dfrac{yze^{xyz} - 2xz^3}{xye^{xyz} - 4yz^3 - 3x^2z^2} = \dfrac{2xz^3 - yze^{xyz}}{xye^{xyz} - 4yz^3 - 3x^2z^2}$ and

$\dfrac{\partial z}{\partial y} = -\dfrac{F_y}{F_z} = -\dfrac{xze^{xyz} - z^4}{xye^{xyz} - 4yz^3 - 3x^2z^2} = \dfrac{z^4 - xze^{xyz}}{xye^{xyz} - 4yz^3 - 3x^2z^2}.$

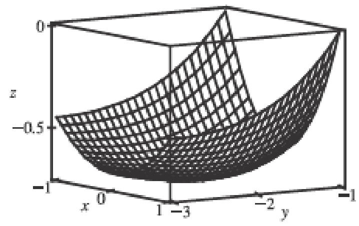
43. $\nabla f = \left\langle z^2\sqrt{y}\,e^{x\sqrt{y}}, \dfrac{xz^2e^{x\sqrt{y}}}{2\sqrt{y}}, 2ze^{x\sqrt{y}}\right\rangle = ze^{x\sqrt{y}}\left\langle z\sqrt{y}, \dfrac{xz}{2\sqrt{y}}, 2\right\rangle$

48. $\nabla f = \langle zye^{xy}, zxe^{xy}, e^{xy}\rangle$, $\nabla f(0,1,2) = \langle 2, 0, 1\rangle$ is the direction of most rapid increase while the rate is $|\langle 2,0,1\rangle| = \sqrt{5}$.

50. The surfaces are $f(x,y,z) = z - 2x^2 + y^2 = 0$ and $g(x,y,z) = z - 4 = 0$. The tangent line is perpendicular to both ∇f and ∇g at $(-2, 2, 4)$. The vector $\mathbf{v} = \nabla f \times \nabla g$ is therefore parallel to the line. $\nabla f(x,y,z) = \langle -4x, 2y, 1\rangle \quad\Rightarrow$ $\nabla f(-2,2,4) = \langle 8,4,1\rangle$, $\nabla g(x,y,z) = \langle 0,0,1\rangle \quad\Rightarrow\quad \nabla g\langle -2,2,4\rangle = \langle 0,0,1\rangle$. Hence

$\mathbf{v} = \nabla f \times \nabla g = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 8 & 4 & 1 \\ 0 & 0 & 1 \end{vmatrix} = 4\,\mathbf{i} - 8\,\mathbf{j}$. Thus, parametric equations are: $x = -2 + 4t,\ y = 2 - 8t,\ z = 4$.

54. $f(x,y) = (x^2+y)e^{y/2} \quad \Rightarrow \quad f_x = 2xe^{y/2}$, $f_y = e^{y/2}(2+x^2+y)/2$, $f_{xx} = 2e^{y/2}$, $f_{yy} = e^{y/2}(4+x^2+y)/4$, $f_{xy} = xe^{y/2}$. Then $f_x = 0$ implies $x = 0$, so $f_y = 0$ implies $y = -2$. But $f_{xx}(0,-2) > 0$, $D(0,-2) = e^{-2} - 0 > 0$ so $f(0,-2) = -2/e$ is a local minimum.



56. Inside D: $f_x = 2xe^{-x^2-y^2}(1-x^2-2y^2) = 0$ implies $x = 0$ or $x^2+2y^2 = 1$. Then if $x = 0$, $f_y = 2ye^{-x^2-y^2}(2-x^2-2y^2) = 0$ implies $y = 0$ or $2-2y^2 = 0$ giving the critical points $(0,0)$, $(0,\pm1)$. If $x^2+2y^2 = 1$, then $f_y = 0$ implies $y = 0$ giving the critical points $(\pm1,0)$. Now $f(0,0) = 0$, $f(\pm1,0) = e^{-1}$ and $f(0,\pm1) = 2e^{-1}$. On the boundary of D: $x^2+y^2 = 4$, so $f(x,y) = e^{-4}(4+y^2)$ and f is smallest when $y = 0$ and largest when $y^2 = 4$. But $f(\pm2,0) = 4e^{-4}$, $f(0,\pm2) = 8e^{-4}$. Thus on D the absolute maximum of f is $f(0,\pm1) = 2e^{-1}$ and the absolute minimum is $f(0,0) = 0$.

60. $f(x,y) = 1/x + 1/y$, $g(x,y) = 1/x^2 + 1/y^2 = 1 \quad \Rightarrow \quad \nabla f = \langle -x^{-2}, -y^{-2} \rangle = \lambda \nabla g = \langle -2\lambda x^{-3}, -2\lambda y^{-3} \rangle$. Then $-x^{-2} = -2\lambda x^3$ or $x = 2\lambda$ and $-y^{-2} = -2\lambda y^{-3}$ or $y = 2\lambda$. Thus $x = y$, so $1/x^2 + 1/y^2 = 2/x^2 = 1$ implies $x = \pm\sqrt{2}$ and the possible points are $(\pm\sqrt{2}, \pm\sqrt{2})$. The absolute maximum of f subject to $x^{-2}+y^{-2} = 1$ is then $f(\sqrt{2},\sqrt{2}) = \sqrt{2}$ and the absolute minimum is $f(-\sqrt{2},-\sqrt{2}) = -\sqrt{2}$.

61. $f(x,y,z) = xyz$, $g(x,y,z) = x^2+y^2+z^2 = 3$. $\nabla f = \lambda \nabla g \quad \Rightarrow \quad \langle yz, xz, xy \rangle = \lambda \langle 2x, 2y, 2z \rangle$. If any of x, y, or z is zero, then $x = y = z = 0$ which contradicts $x^2+y^2+z^2 = 3$. Then $\lambda = \dfrac{yz}{2x} = \dfrac{xz}{2y} = \dfrac{xy}{2z} \quad \Rightarrow \quad 2y^2z = 2x^2z \quad \Rightarrow$ $y^2 = x^2$, and similarly $2yz^2 = 2x^2y \quad \Rightarrow \quad z^2 = x^2$. Substituting into the constraint equation gives $x^2+x^2+x^2 = 3 \quad \Rightarrow$ $x^2 = 1 = y^2 = z^2$. Thus the possible points are $(1,1,\pm1)$, $(1,-1,\pm1)$, $(-1,1,\pm1)$, $(-1,-1,\pm1)$. The absolute maximum is $f(1,1,1) = f(1,-1,-1) = f(-1,1,-1) = f(-1,-1,1) = 1$ and the absolute minimum is $f(1,1,-1) = f(1,-1,1) = f(-1,1,1) = f(-1,-1,-1) = -1$.