6.1

2. $A = \int_0^6 [2x - (x^2 - 4x)]\,dx = \int_0^6 (6x - x^2)\,dx = \left[3x^2 - \frac{1}{3}x^3\right]_0^6 = 108 - 72 = 36$

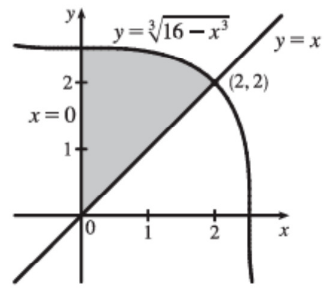
34. The curves intersect when $\sqrt[3]{16 - x^3} = x \;\Rightarrow$

$16 - x^3 = x^3 \;\Rightarrow\; 2x^3 = 16 \;\Rightarrow\; x^3 = 8 \;\Rightarrow\; x = 2.$

Let $f(x) = \sqrt[3]{16 - x^3} - x$ and $\Delta x = \frac{2-0}{4}$.

The shaded area is given by

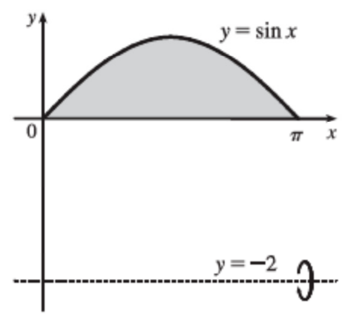
$$A = \int_0^2 f(x)\,dx \approx M_4$$
$$= \tfrac{2}{4}\left[f\left(\tfrac{1}{4}\right) + f\left(\tfrac{3}{4}\right) + f\left(\tfrac{5}{4}\right) + f\left(\tfrac{7}{4}\right)\right]$$
$$\approx 2.8144$$



6.2

34. $V = \pi \int_0^\pi \left\{[\sin x - (-2)]^2 - [0 - (-2)]^2\right\} dx$

$= \pi \int_0^\pi \left[(\sin x + 2)^2 - 2^2\right] dx$



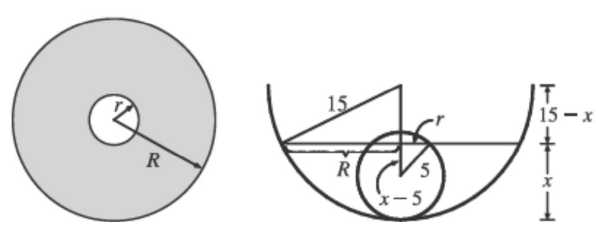
46. $V = \int_0^{10} A(x)\,dx \approx M_5 = \frac{10-0}{5}[A(1) + A(3) + A(5) + A(7) + A(9)]$

$= 2(0.65 + 0.61 + 0.59 + 0.55 + 0.50) = 2(2.90) = 5.80\ \text{m}^3$

68. We consider two cases: one in which the ball is not completely submerged and the other in which it is.

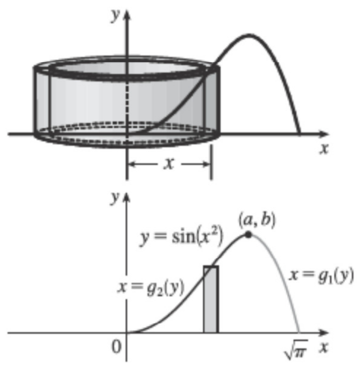
Case 1: $0 \leq h \leq 10$ The ball will not be completely submerged, and so a cross-section of the water parallel to the surface will be the shaded area shown in the first diagram. We can find the area of the cross-section at height x above the bottom of the bowl by using the Pythagorean Theorem: $R^2 = 15^2 - (15 - x)^2$ and $r^2 = 5^2 - (x - 5)^2$, so $A(x) = \pi(R^2 - r^2) = 20\pi x$. The volume of water when it has depth h is then $V(h) = \int_0^h A(x)\,dx = \int_0^h 20\pi x\,dx = \left[10\pi x^2\right]_0^h = 10\pi h^2\ \text{cm}^3$, $0 \leq h \leq 10$.

Case 2: $10 < h \leq 15$ In this case we can find the volume by simply subtracting the volume displaced by the ball from the total volume inside the bowl underneath the surface of the water. The total volume underneath the surface is just the volume of a cap of the bowl, so we use the formula from Exercise 51: $V_{\text{cap}}(h) = \frac{1}{3}\pi h^2(45 - h)$. The volume of the small sphere is $V_{\text{ball}} = \frac{4}{3}\pi(5)^3 = \frac{500}{3}\pi$, so the total volume is $V_{\text{cap}} - V_{\text{ball}} = \frac{1}{3}\pi(45h^2 - h^3 - 500)\ \text{cm}^3$.



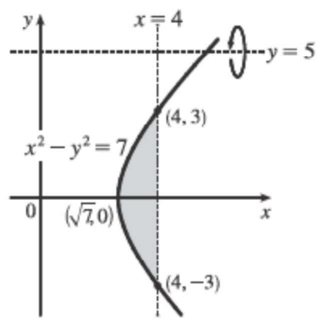
6.3

2.

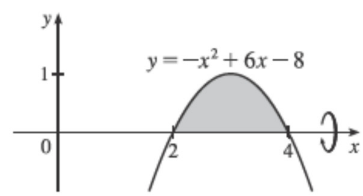


A typical cylindrical shell has circumference $2\pi x$ and height $\sin(x^2)$. $V = \int_0^{\sqrt{\pi}} 2\pi x \sin(x^2)\, dx$. Let $u = x^2$. Then $du = 2x\, dx$, so $V = \pi \int_0^{\pi} \sin u\, du = \pi[-\cos u]_0^{\pi} = \pi[1 - (-1)] = 2\pi$. For slicing, we would first have to locate the local maximum point (a, b) of $y = \sin(x^2)$ using the methods of Chapter 4. Then we would have to solve the equation $y = \sin(x^2)$ for x in terms of y to obtain the functions $x = g_1(y)$ and $x = g_2(y)$ shown in the second figure. Finally we would find the volume using $V = \pi \int_0^b \{[g_1(y)]^2 - [g_2(y)]^2\}\, dy$. Using shells is definitely preferable to slicing.

26. $V = \int_{-3}^{3} 2\pi(5 - y)\left(4 - \sqrt{y^2 + 7}\right) dy$



38. Use disks:



$$V = \int_2^4 \pi(-x^2 + 6x - 8)^2\, dx$$
$$= \pi \int_2^4 (x^4 - 12x^3 + 52x^2 - 96x + 64)\, dx$$
$$= \pi \left[\tfrac{1}{5}x^5 - 3x^4 + \tfrac{52}{3}x^3 - 48x^2 + 64x\right]_2^4$$
$$= \pi\left(\tfrac{512}{15} - \tfrac{496}{15}\right) = \tfrac{16}{15}\pi$$

6.4

2. $W = Fd = (100)(200) = 20{,}000$ ft-lb

6. $W = \int_4^{20} f(x)\, dx \approx M_4 = \Delta x[f(6) + f(10) + f(14) + f(18)] = \frac{20-4}{4}[5.8 + 8.8 + 8.2 + 5.2] = 4(28) = 112$ J

16. The work needed to lift the bucket itself is 4 lb · 80 ft = 320 ft-lb. At time t (in seconds) the bucket is $x_i^* = 2t$ ft above its original 80 ft depth, but it now holds only $(40 - 0.2t)$ lb of water. In terms of distance, the bucket holds $\left[40 - 0.2\left(\tfrac{1}{2}x_i^*\right)\right]$ lb of water when it is x_i^* ft above its original 80 ft depth. Moving this amount of water a distance Δx requires $\left(40 - \tfrac{1}{10}x_i^*\right)\Delta x$ ft-lb of work. Thus, the work needed to lift the water is

$$W = \lim_{n\to\infty} \sum_{i=1}^{n} \left(40 - \tfrac{1}{10}x_i^*\right)\Delta x = \int_0^{80}\left(40 - \tfrac{1}{10}x\right)dx = \left[40x - \tfrac{1}{20}x^2\right]_0^{80} = (3200 - 320) \text{ ft-lb}$$

Adding the work of lifting the bucket gives a total of 3200 ft-lb of work.

22. Let y measure depth (in meters) below the center of the spherical tank, so that $y = -3$ at the top of the tank and $y = -4$ at the spigot. A horizontal disk-shaped "slice" of water Δy m thick and lying at coordinate y has radius $\sqrt{9 - y^2}$ m and volume $\pi r^2 \Delta y = \pi(9 - y^2)\,\Delta y$ m^3. It weighs about $(9.8 \times 1000)\pi(9 - y^2)\,\Delta y$ N must be lifted $(y + 4)$ m by the pump, so the work needed to pump it out is about $(9.8 \times 10^3)(y + 4)\pi(9 - y^2)\,\Delta y$ J. The total work required is

$$W \approx \int_{-3}^{3} (9.8 \times 10^3)(y + 4)\pi(9 - y^2)\,dy = (9.8 \times 10^3)\pi \int_{-3}^{3} (9y - y^3 + 36 - 4y^2)\,dy$$

$$= (9.8 \times 10^3)\pi(2)(4) \int_0^3 (9 - y^2)\,dy \qquad \text{[by Theorem 5.5.6]}$$

$$= (78.4 \times 10^3)\pi \left[9y - \tfrac{1}{3}y^3\right]_0^3 = (78.4 \times 10^3)\pi(18) = 1{,}411{,}200\pi \approx 4.43 \times 10^6 \text{ J}$$

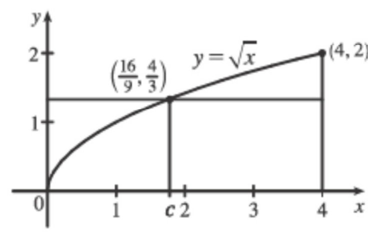
6.5

10. (a) $f_{\text{ave}} = \dfrac{1}{4 - 0} \displaystyle\int_0^4 \sqrt{x}\,dx = \tfrac{1}{4}\left[\tfrac{2}{3}x^{3/2}\right]_0^4$

$= \tfrac{1}{6}\left[x^{3/2}\right]_0^4 = \tfrac{1}{6}[8 - 0] = \tfrac{4}{3}$

(b) $f(c) = f_{\text{ave}} \Leftrightarrow \sqrt{c} = \tfrac{4}{3} \Leftrightarrow c = \tfrac{16}{9}$

(c)



16. (a) $v_{\text{ave}} = \frac{1}{12 - 0}\int_0^{12} v(t)\,dt = \frac{1}{12}I$. Use the Midpoint Rule with $n = 3$ and $\Delta t = \frac{12 - 0}{3} = 4$ to estimate I.

$I \approx M_3 = 4[v(2) + v(6) + v(10)] = 4[21 + 50 + 66] = 4(137) = 548$. Thus, $v_{\text{ave}} \approx \frac{1}{12}(548) = 45\frac{2}{3}$ km/h.

(b) Estimating from the graph, $v(t) = 45\frac{2}{3}$ when $t \approx 5.2$ s.

22. $v_{\text{ave}} = \dfrac{1}{R - 0}\displaystyle\int_0^R v(r)\,dr = \dfrac{1}{R}\int_0^R \dfrac{P}{4\eta l}(R^2 - r^2)\,dr = \dfrac{P}{4\eta l R}\left[R^2 r - \tfrac{1}{3}r^3\right]_0^R = \dfrac{P}{4\eta l R}\left(\tfrac{2}{3}\right)R^3 = \dfrac{PR^2}{6\eta l}$.

Since $v(r)$ is decreasing on $(0, R]$, $v_{\max} = v(0) = \dfrac{PR^2}{4\eta l}$. Thus, $v_{\text{ave}} = \frac{2}{3}v_{\max}$.