5.1

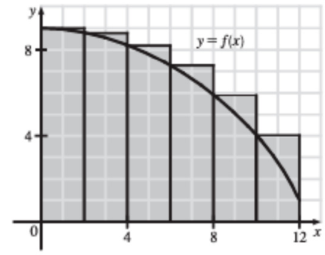
2. (a) (i) $L_6 = \sum_{i=1}^{6} f(x_{i-1})\Delta x \qquad [\Delta x = \frac{12-0}{6} = 2]$

$= 2[f(x_0) + f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5)]$

$= 2[f(0) + f(2) + f(4) + f(6) + f(8) + f(10)]$

$\approx 2(9 + 8.8 + 8.2 + 7.3 + 5.9 + 4.1)$

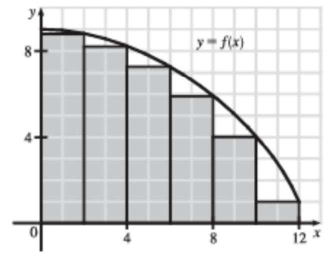
$= 2(43.3) = 86.6$



(ii) $R_6 = L_6 + 2 \cdot f(12) - 2 \cdot f(0)$

$\approx 86.6 + 2(1) - 2(9) = 70.6$

[Add area of rightmost lower rectangle
and subtract area of leftmost upper rectangle.]

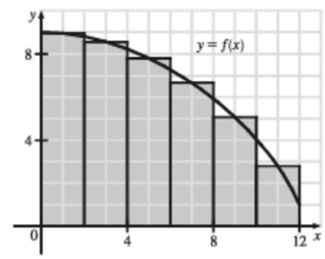


(iii) $M_6 = \sum_{i=1}^{6} f(x_i^*)\,\Delta x$

$= 2[f(1) + f(3) + f(5) + f(7) + f(9) + f(11)]$

$\approx 2(8.9 + 8.5 + 7.8 + 6.6 + 5.1 + 2.8)$

$= 2(39.7) = 79.4$



(b) Since f is *decreasing*, we obtain an *overestimate* by using *left* endpoints; that is, L_6.

(c) Since f is *decreasing*, we obtain an *underestimate* by using *right* endpoints; that is, R_6.

(d) M_6 gives the best estimate, since the area of each rectangle appears to be closer to the true area than the overestimates and underestimates in L_6 and R_6.

18. $f(x) = 1 + x^4,\ \ 2 \le x \le 5.\quad \Delta x = (5-2)/n = 3/n$ and $x_i = 2 + i\,\Delta x = 2 + 3i/n$.

$$A = \lim_{n\to\infty} R_n = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x = \lim_{n\to\infty} \sum_{i=1}^{n} \left[1 + \left(2 + \frac{3i}{n}\right)^4\right] \cdot \frac{3}{n}.$$

5.2

8. (a) Using the right endpoints to approximate $\int_3^9 f(x)\,dx$, we have

$$\sum_{i=1}^{3} f(x_i)\,\Delta x = 2[f(5) + f(7) + f(9)] = 2\,(-0.6 + 0.9 + 1.8) = 4.2.$$

Since f is *increasing*, using *right* endpoints gives an *overestimate*.

(b) Using the left endpoints to approximate $\int_3^9 f(x)\,dx$, we have

$$\sum_{i=1}^{3} f(x_{i-1})\,\Delta x = 2[f(3) + f(5) + f(7)] = 2\,(-3.4 - 0.6 + 0.9) = -6.2.$$

Since f is *increasing*, using *left* endpoints gives an *underestimate*.

(c) Using the midpoint of each interval to approximate $\int_3^9 f(x)\,dx$, we have

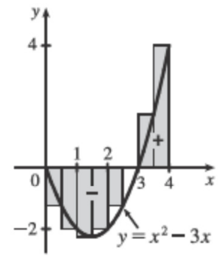
$$\sum_{i=1}^{3} f(\overline{x}_i)\,\Delta x = 2[f(4) + f(6) + f(8)] = 2\,(-2.1 + 0.3 + 1.4) = -0.8.$$

We cannot say anything about the midpoint estimate compared to the exact value of the integral.

26. (a) $\Delta x = (4-0)/8 = 0.5$ and $x_i^* = x_i = 0.5i$.

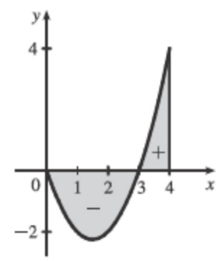
$$\int_0^4 (x^2-3x)\,dx \approx \sum_{i=1}^{8} f(x_i^*)\,\Delta x$$
$$= 0.5\{[0.5^2 - 3(0.5)] + [1.0^2 - 3(1.0)] + \cdots + [3.5^2 - 3(3.5)] + [4.0^2 - 3(4.0)]\}$$
$$= \tfrac{1}{2}\left(-\tfrac{5}{4} - 2 - \tfrac{9}{4} - 2 - \tfrac{5}{4} + 0 + \tfrac{7}{4} + 4\right) = -1.5$$

(b)

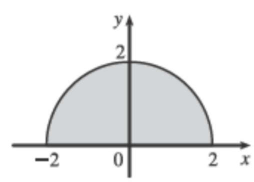


(c) $$\int_0^4 (x^2-3x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(\frac{4i}{n}\right)^2 - 3\left(\frac{4i}{n}\right)\right]\left(\frac{4}{n}\right)$$
$$= \lim_{n\to\infty} \frac{4}{n}\left[\frac{16}{n^2}\sum_{i=1}^{n} i^2 - \frac{12}{n}\sum_{i=1}^{n} i\right]$$
$$= \lim_{n\to\infty} \left[\frac{64}{n^3}\cdot\frac{n(n+1)(2n+1)}{6} - \frac{48}{n^2}\cdot\frac{n(n+1)}{2}\right]$$
$$= \lim_{n\to\infty} \left[\frac{32}{3}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right) - 24\left(1+\frac{1}{n}\right)\right]$$
$$= \tfrac{32}{3}\cdot 2 - 24 = -\tfrac{8}{3}$$

(d) $\int_0^4 (x^2-3x)\,dx = A_1 - A_2$, where A_1 is the area marked $+$ and A_2 is the area marked $-$.



36. $\int_{-2}^{2} \sqrt{4-x^2}\,dx$ can be interpreted as the area under the graph of $f(x) = \sqrt{4-x^2}$ between $x=-2$ and $x=2$. This is equal to half the area of the circle with radius 2, so $\int_{-2}^{2} \sqrt{4-x^2}\,dx = \frac{1}{2}\pi\cdot 2^2 = 2\pi$.



5.3

30. $\int_0^2 (y-1)(2y+1)\,dy = \int_0^2 (2y^2-y-1)\,dy = \left[\frac{2}{3}y^3 - \frac{1}{2}y^2 - y\right]_0^2 = \left(\frac{16}{3} - 2 - 2\right) - 0 = \frac{4}{3}$

32. $\int_0^{\pi/4} \sec\theta\,\tan\theta\,d\theta = [\sec\theta]_0^{\pi/4} = \sec\frac{\pi}{4} - \sec 0 = \sqrt{2} - 1$

52. For the curve to be concave upward, we must have $y'' > 0$.

$y = \displaystyle\int_0^x \frac{1}{1+t+t^2}\,dt \;\Rightarrow\; y' = \frac{1}{1+x+x^2} \;\Rightarrow\; y'' = \frac{-(1+2x)}{(1+x+x^2)^2}$. For this expression to be positive, we must have $(1+2x) < 0$, since $(1+x+x^2)^2 > 0$ for all x. $(1+2x) < 0 \;\Leftrightarrow\; x < -\frac{1}{2}$. Thus, the curve is concave upward on $\left(-\infty, -\frac{1}{2}\right)$.

5.4

2. $\dfrac{d}{dx}[x\sin x + \cos x + C] = x\cos x + (\sin x)\cdot 1 - \sin x + 0 = x\cos x$

16. $\displaystyle\int \frac{\sin 2x}{\sin x}\,dx = \int \frac{2\sin x\,\cos x}{\sin x}\,dx = \int 2\cos x\,dx = 2\sin x + C$

66. (a) From Exercise 4.1.66(a), $v(t) = 0.00146t^3 - 0.11553t^2 + 24.98169t - 21.26872$.

(b) $h(125) - h(0) = \int_0^{125} v(t)\,dt = \left[0.000365t^4 - 0.03851t^3 + 12.490845t^2 - 21.26872t\right]_0^{125} \approx 206{,}407$ ft

5.5

2. Let $u = 2 + x^4$. Then $du = 4x^3\,dx$ and $x^3\,dx = \frac{1}{4}\,du$,

so $\displaystyle\int x^3(2+x^4)^5\,dx = \int u^5\left(\tfrac{1}{4}\,du\right) = \frac{1}{4}\frac{u^6}{6} + C = \tfrac{1}{24}(2+x^4)^6 + C.$

10. Let $u = 3t + 2$. Then $du = 3\,dt$ and $dt = \frac{1}{3}\,du$, so

$$\int (3t+2)^{2.4}\,dt = \int u^{2.4}\left(\tfrac{1}{3}\,du\right) = \frac{1}{3}\frac{u^{3.4}}{3.4} + C = \frac{1}{10.2}(3t+2)^{3.4} + C.$$

42. $\displaystyle\int_{-\pi/2}^{\pi/2} \frac{x^2 \sin x}{1+x^6}\,dx = 0$ by Theorem 6(b), since $f(x) = \dfrac{x^2 \sin x}{1+x^6}$ is an odd function.