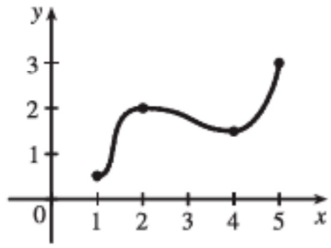
4.1

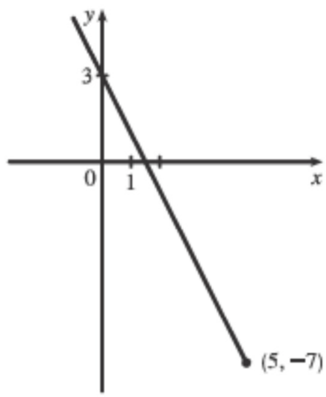
8. Absolute minimum at 1, absolute maximum at 5,

local maximum at 2, local minimum at 4



16. $f(x) = 3 - 2x$, $x \le 5$. Absolute minimum $f(5) = -7$;

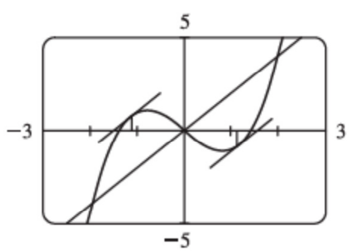
no local minimum. No absolute or local maximum.



68. $g(x) = 2 + (x - 5)^3 \;\Rightarrow\; g'(x) = 3(x - 5)^2 \;\Rightarrow\; g'(5) = 0$, so 5 is a critical number. But $g(5) = 2$ and g takes on values > 2 and values < 2 in any open interval containing 5, so g does not have a local maximum or minimum at 5.

4.2

10. (a)



It seems that the tangent lines are parallel to the secant at $x \approx \pm 1.2$.

(b) The slope of the secant line is 2, and its equation is $y = 2x$. $f(x) = x^3 - 2x \;\Rightarrow\; f'(x) = 3x^2 - 2$, so we solve $f'(c) = 2 \;\Rightarrow\; 3c^2 = 4 \;\Rightarrow$ $c = \pm\frac{2\sqrt{3}}{3} \approx 1.155$. Our estimates were off by about 0.045 in each case.

18. Let $f(x) = 2x - 1 - \sin x$. Then $f(0) = -1 < 0$ and $f(\pi/2) = \pi - 2 > 0$. f is the sum of the polynomial $2x - 1$ and the scalar multiple $(-1) \cdot \sin x$ of the trigonometric function $\sin x$, so f is continuous (and differentiable) for all x. By the Intermediate Value Theorem, there is a number c in $(0, \pi/2)$ such that $f(c) = 0$. Thus, the given equation has at least one real root. If the equation has distinct real roots a and b with $a < b$, then $f(a) = f(b) = 0$. Since f is continuous on $[a, b]$ and differentiable on (a, b), Rolle's Theorem implies that there is a number r in (a, b) such that $f'(r) = 0$. But $f'(r) = 2 - \cos r > 0$ since $\cos r \le 1$. This contradiction shows that the given equation can't have two distinct real roots, so it has exactly one real root.

32. Let $v(t)$ be the velocity of the car t hours after 2:00 PM. Then $\dfrac{v(1/6)-v(0)}{1/6-0}=\dfrac{50-30}{1/6}=120$. By the Mean Value Theorem, there is a number c such that $0<c<\frac{1}{6}$ with $v'(c)=120$. Since $v'(t)$ is the acceleration at time t, the acceleration c hours after 2:00 PM is exactly $120\ \mathrm{mi/h^2}$.

4.3

2. (a) f is increasing on $(0,1)$ and $(3,7)$.

(b) f is decreasing on $(1,3)$.

(c) f is concave upward on $(2,4)$ and $(5,7)$.

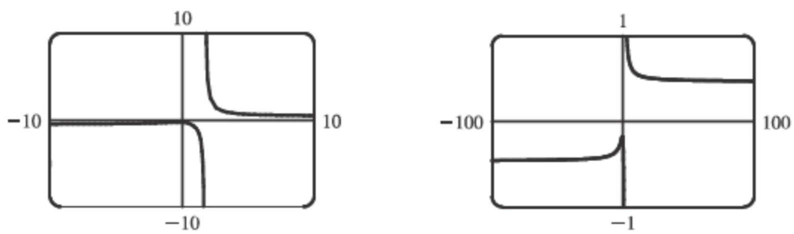
(d) f is concave downward on $(0,2)$ and $(4,5)$.

(e) The points of inflection are $(2,2)$, $(4,3)$, and $(5,4)$.

4.4

22. $\lim\limits_{x\to\infty}\cos x$ does not exist because as x increases $\cos x$ does not approach any one value, but oscillates between 1 and -1.

40. (a)



From the graph, it appears at first that there is only one horizontal asymptote, at $y\approx 0$, and a vertical asymptote at $x\approx 1.7$. However, if we graph the function with a wider viewing rectangle, we see that in fact there seem to be two horizontal asymptotes: one at $y\approx 0.5$ and one at $y\approx -0.5$. So we estimate that

$$\lim_{x\to\infty}\frac{\sqrt{2x^2+1}}{3x-5}\approx 0.5 \qquad\text{and}\qquad \lim_{x\to-\infty}\frac{\sqrt{2x^2+1}}{3x-5}\approx -0.5$$

(b) $f(1000)\approx 0.4722$ and $f(10{,}000)\approx 0.4715$, so we estimate that $\lim\limits_{x\to\infty}\dfrac{\sqrt{2x^2+1}}{3x-5}\approx 0.47$.

$f(-1000)\approx -0.4706$ and $f(-10{,}000)\approx -0.4713$, so we estimate that $\lim\limits_{x\to-\infty}\dfrac{\sqrt{2x^2+1}}{3x-5}\approx -0.47$.

(c) $\lim\limits_{x\to\infty}\dfrac{\sqrt{2x^2+1}}{3x-5}=\lim\limits_{x\to\infty}\dfrac{\sqrt{2+1/x^2}}{3-5/x}$ [since $\sqrt{x^2}=x$ for $x>0$] $=\dfrac{\sqrt{2}}{3}\approx 0.471404$.

For $x<0$, we have $\sqrt{x^2}=|x|=-x$, so when we divide the numerator by x, with $x<0$, we get $\dfrac{1}{x}\sqrt{2x^2+1}=-\dfrac{1}{\sqrt{x^2}}\sqrt{2x^2+1}=-\sqrt{2+1/x^2}$. Therefore,

$$\lim_{x\to-\infty}\frac{\sqrt{2x^2+1}}{3x-5}=\lim_{x\to-\infty}\frac{-\sqrt{2+1/x^2}}{3-5/x}=-\frac{\sqrt{2}}{3}\approx -0.471404.$$

60. (a) After t minutes, $25t$ liters of brine with 30 g of salt per liter has been pumped into the tank, so it contains $(5000+25t)$ liters of water and $25t\cdot 30=750t$ grams of salt. Therefore, the salt concentration at time t will be $C(t)=\dfrac{750t}{5000+25t}=\dfrac{30t}{200+t}\ \dfrac{\mathrm{g}}{\mathrm{L}}$.

(b) $\lim\limits_{t\to\infty}C(t)=\lim\limits_{t\to\infty}\dfrac{30t}{200+t}=\lim\limits_{t\to\infty}\dfrac{30t/t}{200/t+t/t}=\dfrac{30}{0+1}=30$. So the salt concentration approaches that of the brine being pumped into the tank.

4.5

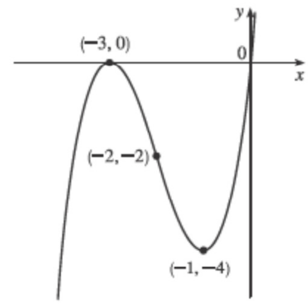
2. $y = f(x) = x^3 + 6x^2 + 9x = x(x+3)^2$ **A.** $D = \mathbb{R}$ **B.** x-intercepts are -3 and 0, y-intercept $= 0$ **C.** No symmetry **D.** No asymptote

E. $f'(x) = 3x^2 + 12x + 9 = 3(x+1)(x+3) < 0 \quad \Leftrightarrow \quad -3 < x < -1$, so f is decreasing on $(-3, -1)$ and increasing on $(-\infty, -3)$ and $(-1, \infty)$.

F. Local maximum value $f(-3) = 0$, local minimum value $f(-1) = -4$

G. $f''(x) = 6x + 12 = 6(x+2) > 0 \quad \Leftrightarrow \quad x > -2$, so f is CU on $(-2, \infty)$ and CD on $(-\infty, -2)$. IP at $(-2, -2)$

H.



40. Let $a = m_0^2 c^4$ and $b = h^2 c^2$, so the equation can be written as $E = f(\lambda) = \sqrt{a + b/\lambda^2} = \sqrt{\dfrac{a\lambda^2 + b}{\lambda^2}} = \dfrac{\sqrt{a\lambda^2 + b}}{\lambda}$.

$\displaystyle\lim_{\lambda \to 0^+} \frac{\sqrt{a\lambda^2 + b}}{\lambda} = \infty$, so $\lambda = 0$ is a VA.

$\displaystyle\lim_{\lambda \to \infty} \frac{\sqrt{a\lambda^2 + b}}{\lambda} = \lim_{\lambda \to \infty} \frac{\sqrt{a\lambda^2 + b}/\lambda}{\lambda/\lambda} = \lim_{\lambda \to \infty} \frac{\sqrt{a + b/\lambda^2}}{1} = \sqrt{a}$, so $E = \sqrt{a} = m_0 c^2$ is a HA.

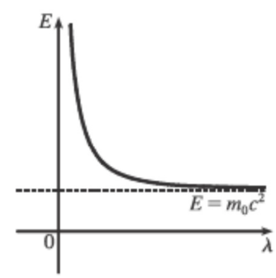
$$f'(\lambda) = \frac{\lambda \cdot \frac{1}{2}(a\lambda^2 + b)^{-1/2}(2a\lambda) - (a\lambda^2 + b)^{1/2}(1)}{\lambda^2} = \frac{(a\lambda^2 + b)^{-1/2}[a\lambda^2 - (a\lambda^2 + b)]}{\lambda^2} = \frac{-b}{\lambda^2\sqrt{a\lambda^2 + b}} < 0,$$

so f is decreasing on $(0, \infty)$. Using the Reciprocal Rule,

$$f''(\lambda) = b \cdot \frac{\lambda^2 \cdot \frac{1}{2}(a\lambda^2 + b)^{-1/2}(2a\lambda) + (a\lambda^2 + b)^{1/2}(2\lambda)}{\left(\lambda^2\sqrt{a\lambda^2 + b}\right)^2}$$

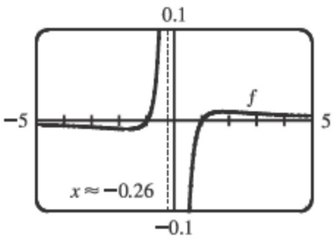
$$= \frac{b\lambda(a\lambda^2 + b)^{-1/2}[a\lambda^2 + 2(a\lambda^2 + b)]}{\left(\lambda^2\sqrt{a\lambda^2 + b}\right)^2} = \frac{b(3a\lambda^2 + 2b)}{\lambda^3(a\lambda^2 + b)^{3/2}} > 0,$$

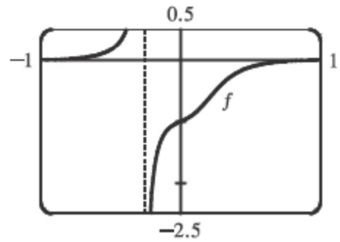
so f is CU on $(0, \infty)$. There are no extrema or inflection points. The graph shows that as λ decreases, the energy increases and as λ increases, the energy decreases. For large wavelengths, the energy is very close to the energy at rest.

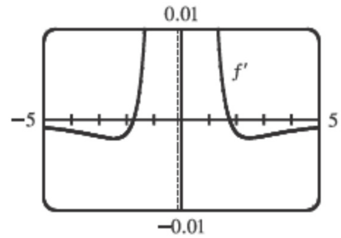


4.6

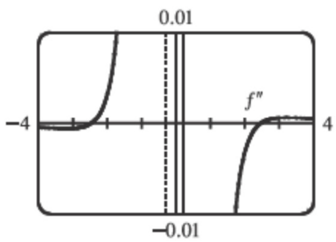
4. $f(x) = \dfrac{x^2 - 1}{40x^3 + x + 1} \;\;\Rightarrow\;\; f'(x) = \dfrac{-40x^4 + 121x^2 + 2x + 1}{(40x^3 + x + 1)^2} \;\;\Rightarrow\;\; f''(x) = \dfrac{80x(40x^5 - 243x^3 - 7x^2 - 3x + 3)}{(40x^3 + x + 1)^3}$

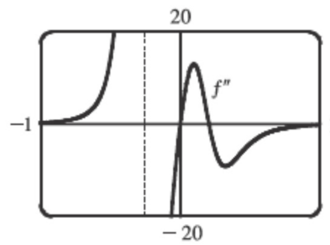






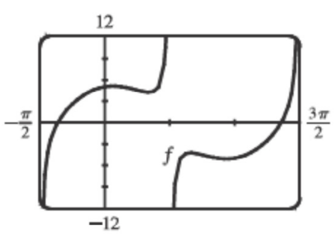
From the first graph of f, we see that there is a VA at $x \approx -0.26$. From the graph of f', we estimate that f is decreasing on $(-\infty, -1.73)$, increasing on $(-1.73, -0.26)$, increasing on $(-0.26, 1.75)$, and decreasing on $(1.75, \infty)$, with local minimum value $f(-1.73) \approx -0.01$ and local maximum value $f(1.75) \approx 0.01$.

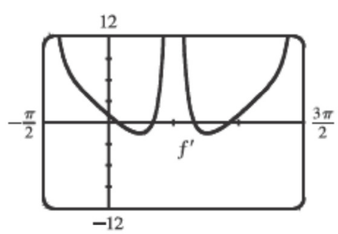


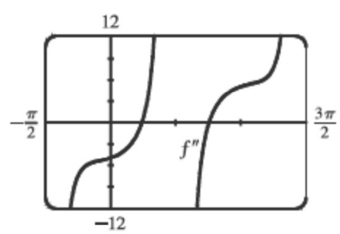


From the graphs of f'', we estimate that f is CD on $(-\infty, -2.45)$, CU on $(-2.45, -0.26)$, CD on $(-0.26, 0)$, CU on $(0, 0.21)$, CD on $(0.21, 2.48)$, and CU on $(2.48, \infty)$. There is an inflection point at $(0, -1)$ and at about $(-2.45, -0.01)$, $(0.21, -0.62)$, and $(2.48, 0.00)$.

6. $f(x) = \tan x + 5\cos x \;\;\Rightarrow\;\; f'(x) = \sec^2 x - 5\sin x \;\;\Rightarrow\;\; f''(x) = 2\sec^2 x\,\tan x - 5\,\cos x$. Since f is periodic with period 2π, and defined for all x except odd multiples of $\frac{\pi}{2}$, we graph f and its derivatives on $\left[-\frac{\pi}{2}, \frac{3\pi}{2}\right]$.





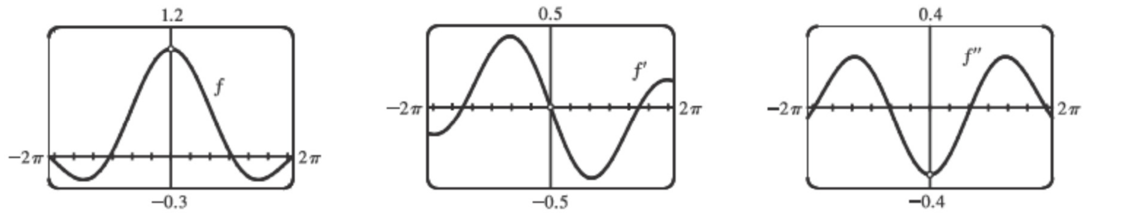


We estimate from the graph of f' that f is increasing on $\left(-\frac{\pi}{2}, 0.21\right)$, $\left(1.07, \frac{\pi}{2}\right)$, $\left(\frac{\pi}{2}, 2.07\right)$, and $\left(2.93, \frac{3\pi}{2}\right)$, and decreasing on and $(2.07, 2.93)$. Local minimum values: $f(1.07) \approx 4.23$, $f(2.93) \approx -5.10$. Local maximum values: $f(0.21) \approx 5.10$, $f(2.07) \approx -4.23$.

From the graph of f'', we estimate that f is CU on $\left(0.76, \frac{\pi}{2}\right)$ and $\left(2.38, \frac{3\pi}{2}\right)$, and CD on $\left(-\frac{\pi}{2}, 0.76\right)$ and $\left(\frac{\pi}{2}, 2.38\right)$. f has IP at $(0.76, 4.57)$ and $(2.38, -4.57)$.

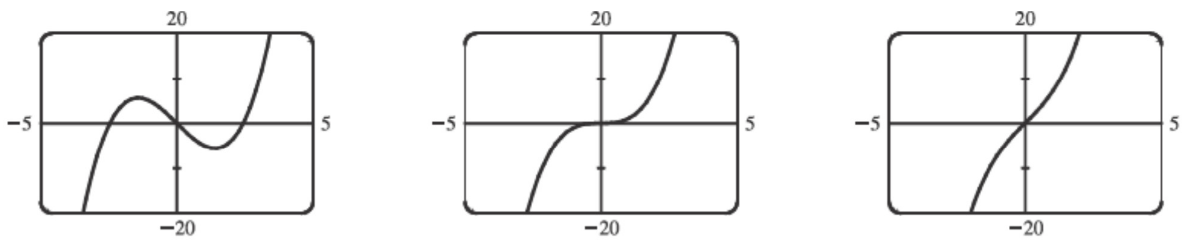
8. $f(x) = \dfrac{\sin x}{x}$, $-2\pi \le x \le 2\pi$. $f'(x) = \dfrac{x\cos x - \sin x}{x^2} \Rightarrow$

$$f''(x) = \frac{x^2(\cos x - x\sin x - \cos x) - (x\cos x - \sin x)(2x)}{(x^2)^2} = \frac{-x^2\sin x - 2x\cos x + 2\sin x}{x^3}$$



f is an even function with domain $(-\infty, 0) \cup (0, \infty)$. There is no y-intercept, but $\lim_{x\to 0} f(x) = 1$. The x-intercepts are -2π, $-\pi$, 0, π, and 2π. From the graph of f', we estimate that f is decreasing on $(-2\pi, -4.49)$, increasing on $(-4.49, 0)$, decreasing on $(0, 4.49)$, and increasing on $(4.49, 2\pi)$. Thus, f has local minima of $f(\pm 4.49) \approx -0.22$. From the graph of f'', we estimate that f is CD on $(-2\pi, -5.94)$, CU on $(-5.94, -2.08)$, CD on $(-2.08, 0)$ and $(0, 2.08)$, CU on $(2.08, 5.94)$, and CD on $(5.94, 2\pi)$. f has IPs at approximately $(\pm 5.94, -0.06)$ and $(\pm 2.08, 0.42)$.

20. $f(x) = x^3 + cx = x(x^2 + c) \Rightarrow f'(x) = 3x^2 + c \Rightarrow f''(x) = 6x$



$c = -6$ $c = 0$ $c = 6$

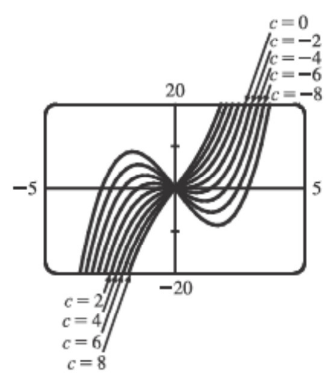
x-intercepts: When $c \ge 0$, 0 is the only x-intercept. When $c < 0$, the x-intercepts are 0 and $\pm\sqrt{-c}$.

y-intercept $= f(0) = 0$. f is odd, so the graph is symmetric with respect to the origin. $f''(x) < 0$ for $x < 0$ and $f''(x) > 0$ for $x > 0$, so f is CD on $(-\infty, 0)$ and CU on $(0, \infty)$. The origin is the only inflection point.

If $c > 0$, then $f'(x) > 0$ for all x, so f is increasing and has no local maximum or minimum.

If $c = 0$, then $f'(x) \ge 0$ with equality at $x = 0$, so again f is increasing and has no local maximum or minimum.

If $c < 0$, then $f'(x) = 3[x^2 - (-c/3)] = 3\left(x + \sqrt{-c/3}\right)\left(x - \sqrt{-c/3}\right)$, so $f'(x) > 0$ on $\left(-\infty, -\sqrt{-c/3}\right)$ and $\left(\sqrt{-c/3}, \infty\right)$; $f'(x) < 0$ on $\left(-\sqrt{-c/3}, \sqrt{-c/3}\right)$. It follows that $f\left(-\sqrt{-c/3}\right) = -\frac{2}{3}c\sqrt{-c/3}$ is a local maximum value and $f\left(\sqrt{-c/3}\right) = \frac{2}{3}c\sqrt{-c/3}$ is a local minimum value. As c decreases (toward more negative values), the local maximum and minimum move further apart.



There is no absolute maximum or minimum value. The only transitional value of c corresponding to a change in character of the graph is $c = 0$.

4.7

8. We need to maximize P for $I \geq 0$. $P(I) = \dfrac{100I}{I^2 + I + 4} \Rightarrow$

$$P'(I) = \frac{(I^2+I+4)(100) - 100I(2I+1)}{(I^2+I+4)^2} = \frac{100(I^2+I+4-2I^2-I)}{(I^2+I+4)^2} = \frac{-100(I^2-4)}{(I^2+I+4)^2} = \frac{-100(I+2)(I-2)}{(I^2+I+4)^2}.$$

$P'(I) > 0$ for $0 < I < 2$ and $P'(I) < 0$ for $I > 2$. Thus, P has an absolute maximum of $P(2) = 20$ at $I = 2$.

16. (a) Let the rectangle have sides x and y and area A, so $A = xy$ or $y = A/x$. The problem is to minimize the perimeter $= 2x + 2y = 2x + 2A/x = P(x)$. Now $P'(x) = 2 - 2A/x^2 = 2(x^2 - A)/x^2$. So the critical number is $x = \sqrt{A}$. Since $P'(x) < 0$ for $0 < x < \sqrt{A}$ and $P'(x) > 0$ for $x > \sqrt{A}$, there is an absolute minimum at $x = \sqrt{A}$. The sides of the rectangle are $\sqrt{A}$ and $A/\sqrt{A} = \sqrt{A}$, so the rectangle is a square.

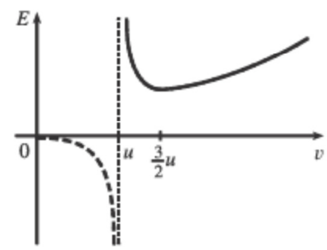
(b) Let p be the perimeter and x and y the lengths of the sides, so $p = 2x + 2y \Rightarrow 2y = p - 2x \Rightarrow y = \frac{1}{2}p - x$. The area is $A(x) = x\left(\frac{1}{2}p - x\right) = \frac{1}{2}px - x^2$. Now $A'(x) = 0 \Rightarrow \frac{1}{2}p - 2x = 0 \Rightarrow 2x = \frac{1}{2}p \Rightarrow x = \frac{1}{4}p$. Since $A''(x) = -2 < 0$, there is an absolute maximum for A when $x = \frac{1}{4}p$ by the Second Derivative Test. The sides of the rectangle are $\frac{1}{4}p$ and $\frac{1}{2}p - \frac{1}{4}p = \frac{1}{4}p$, so the rectangle is a square.

42. (a) $E(v) = \dfrac{aLv^3}{v-u} \Rightarrow E'(v) = aL\dfrac{(v-u)3v^2 - v^3}{(v-u)^2} = 0$ when

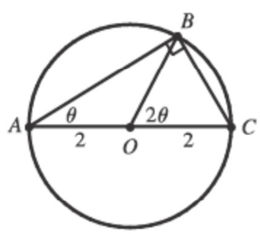
$2v^3 = 3uv^2 \Rightarrow 2v = 3u \Rightarrow v = \frac{3}{2}u$.

The First Derivative Test shows that this value of v gives the minimum value of E.

(b)



46.



In isosceles triangle AOB, $\angle O = 180° - \theta - \theta$, so $\angle BOC = 2\theta$. The distance rowed is $4\cos\theta$ while the distance walked is the length of arc $BC = 2(2\theta) = 4\theta$. The time taken is given by $T(\theta) = \dfrac{4\cos\theta}{2} + \dfrac{4\theta}{4} = 2\cos\theta + \theta$, $0 \leq \theta \leq \frac{\pi}{2}$.

$T'(\theta) = -2\sin\theta + 1 = 0 \Leftrightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$.

Check the value of T at $\theta = \frac{\pi}{6}$ and at the endpoints of the domain of T; that is, $\theta = 0$ and $\theta = \frac{\pi}{2}$.
$T(0) = 2$, $T\left(\frac{\pi}{6}\right) = \sqrt{3} + \frac{\pi}{6} \approx 2.26$, and $T\left(\frac{\pi}{2}\right) = \frac{\pi}{2} \approx 1.57$. Therefore, the minimum value of T is $\frac{\pi}{2}$ when $\theta = \frac{\pi}{2}$; that is, the woman should walk all the way. Note that $T''(\theta) = -2\cos\theta < 0$ for $0 \leq \theta < \frac{\pi}{2}$, so $\theta = \frac{\pi}{6}$ gives a maximum time.

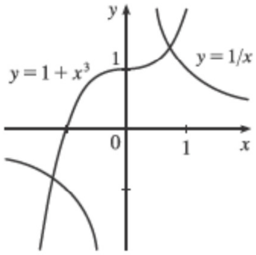
4.8

6. $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 + 3 \Rightarrow f'(x) = x^2 + x$, so $x_{n+1} = x_n - \dfrac{\frac{1}{3}x_n^3 + \frac{1}{2}x_n^2 + 3}{x_n^2 + x_n}$. Now $x_1 = -3 \Rightarrow$

$$x_2 = -3 - \frac{-9 + \frac{9}{2} + 3}{9 - 3} = -3 - \left(-\tfrac{1}{4}\right) = -2.75 \Rightarrow x_3 = -2.75 - \frac{\frac{1}{3}(-2.75)^3 + \frac{1}{2}(-2.75)^2 + 3}{(-2.75)^2 + (-2.75)} \approx -2.7186.$$

12. $f(x) = x^{100} - 100 \Rightarrow f'(x) = 100x^{99}$, so $x_{n+1} = x_n - \dfrac{x_n^{100} - 100}{100x_n^{99}}$. We need to find approximations until they agree to eight decimal places. $x_1 = 1.05 \Rightarrow x_2 \approx 1.04748471$, $x_3 \approx 1.04713448$, $x_4 \approx 1.04712855 \approx x_5$.
So $\sqrt[100]{100} \approx 1.04712855$, to eight decimal places.

20.



From the graph, we see that there appear to be points of intersection near $x = -1.2$ and $x = 0.8$. Solving $\frac{1}{x} = 1 + x^3$ is the same as solving $f(x) = \frac{1}{x} - 1 - x^3 = 0$. $f(x) = \frac{1}{x} - 1 - x^3 \Rightarrow f'(x) = -\frac{1}{x^2} - 3x^2$, so $x_{n+1} = x_n - \frac{1/x_n - 1 - x_n^3}{-1/x_n^2 - 3x_n^2}$.

$x_1 = -1.2$	$x_1 = 0.8$
$x_2 \approx -1.221006$	$x_2 \approx 0.724767$
$x_3 \approx -1.220744 \approx x_4$	$x_3 \approx 0.724492 \approx x_4$

To six decimal places, the roots of the equation are -1.220744 and 0.724492.

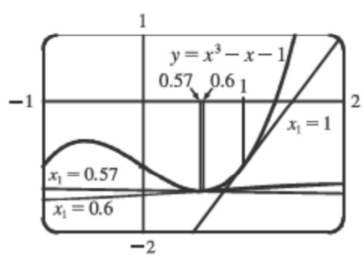
30. $x^3 - x = 1 \Leftrightarrow x^3 - x - 1 = 0$. $f(x) = x^3 - x - 1 \Rightarrow f'(x) = 3x^2 - 1$, so $x_{n+1} = x_n - \frac{x_n^3 - x_n - 1}{3x_n^2 - 1}$.

(a) $x_1 = 1$, $x_2 = 1.5$, $x_3 \approx 1.347826$, $x_4 \approx 1.325200$, $x_5 \approx 1.324718 \approx x_6$

(b) $x_1 = 0.6$, $x_2 = 17.9$, $x_3 \approx 11.946802$, $x_4 \approx 7.985520$, $x_5 \approx 5.356909$, $x_6 \approx 3.624996$, $x_7 \approx 2.505589$, $x_8 \approx 1.820129$, $x_9 \approx 1.461044$, $x_{10} \approx 1.339323$, $x_{11} \approx 1.324913$, $x_{12} \approx 1.324718 \approx x_{13}$

(c) $x_1 = 0.57$, $x_2 \approx -54.165455$, $x_3 \approx -36.114293$, $x_4 \approx -24.082094$, $x_5 \approx -16.063387$, $x_6 \approx -10.721483$, $x_7 \approx -7.165534$, $x_8 \approx -4.801704$, $x_9 \approx -3.233425$, $x_{10} \approx -2.193674$, $x_{11} \approx -1.496867$, $x_{12} \approx -0.997546$, $x_{13} \approx -0.496305$, $x_{14} \approx -2.894162$, $x_{15} \approx -1.967962$, $x_{16} \approx -1.341355$, $x_{17} \approx -0.870187$, $x_{18} \approx -0.249949$, $x_{19} \approx -1.192219$, $x_{20} \approx -0.731952$, $x_{21} \approx 0.355213$, $x_{22} \approx -1.753322$, $x_{23} \approx -1.189420$, $x_{24} \approx -0.729123$, $x_{25} \approx 0.377844$, $x_{26} \approx -1.937872$, $x_{27} \approx -1.320350$, $x_{28} \approx -0.851919$, $x_{29} \approx -0.200959$, $x_{30} \approx -1.119386$, $x_{31} \approx -0.654291$, $x_{32} \approx 1.547010$, $x_{33} \approx 1.360051$, $x_{34} \approx 1.325828$, $x_{35} \approx 1.324719$, $x_{36} \approx 1.324718 \approx x_{37}$.

(d)



From the figure, we see that the tangent line corresponding to $x_1 = 1$ results in a sequence of approximations that converges quite quickly ($x_5 \approx x_6$). The tangent line corresponding to $x_1 = 0.6$ is close to being horizontal, so x_2 is quite far from the root. But the sequence still converges — just a little more slowly ($x_{12} \approx x_{13}$). Lastly, the tangent line corresponding to $x_1 = 0.57$ is very nearly horizontal, x_2 is farther away from the root, and the sequence takes more iterations to converge ($x_{36} \approx x_{37}$).

4.9

42. $f'(x) = x^3 \Rightarrow f(x) = \frac{1}{4}x^4 + C$. $x + y = 0 \Rightarrow y = -x \Rightarrow m = -1$. Now $m = f'(x) \Rightarrow -1 = x^3 \Rightarrow x = -1 \Rightarrow y = 1$ (from the equation of the tangent line), so $(-1, 1)$ is a point on the graph of f. From f, $1 = \frac{1}{4}(-1)^4 + C \Rightarrow C = \frac{3}{4}$. Therefore, the function is $f(x) = \frac{1}{4}x^4 + \frac{3}{4}$.

54. $a(t) = v'(t) = \cos t + \sin t \Rightarrow v(t) = \sin t - \cos t + C \Rightarrow 5 = v(0) = -1 + C \Rightarrow C = 6$, so $v(t) = \sin t - \cos t + 6 \Rightarrow s(t) = -\cos t - \sin t + 6t + D \Rightarrow 0 = s(0) = -1 + D \Rightarrow D = 1$, so $s(t) = -\cos t - \sin t + 6t + 1$.

70. (a) For $0 \le t \le 3$ we have $a(t) = 60t \;\Rightarrow\; v(t) = 30t^2 + C \;\Rightarrow\; v(0) = 0 = C \;\Rightarrow\; v(t) = 30t^2$, so

$s(t) = 10t^3 + C \;\Rightarrow\; s(0) = 0 = C \;\Rightarrow\; s(t) = 10t^3$. Note that $v(3) = 270$ and $s(3) = 270$.

For $3 < t \le 17$: $a(t) = -g = -32$ ft/s $\;\Rightarrow\; v(t) = -32(t-3) + C \;\Rightarrow\; v(3) = 270 = C \;\Rightarrow$

$v(t) = -32(t-3) + 270 \;\Rightarrow\; s(t) = -16(t-3)^2 + 270(t-3) + C \;\Rightarrow\; s(3) = 270 = C \;\Rightarrow$

$s(t) = -16(t-3)^2 + 270(t-3) + 270$. Note that $v(17) = -178$ and $s(17) = 914$.

For $17 < t \le 22$: The velocity increases linearly from -178 ft/s to -18 ft/s during this period, so

$\dfrac{\Delta v}{\Delta t} = \dfrac{-18-(-178)}{22-17} = \dfrac{160}{5} = 32$. Thus, $v(t) = 32(t-17) - 178 \;\Rightarrow$

$s(t) = 16(t-17)^2 - 178(t-17) + 914$ and $s(22) = 424$ ft.

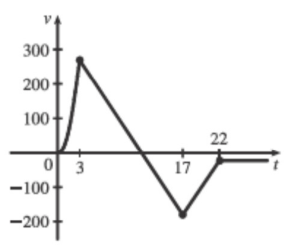
For $t > 22$: $v(t) = -18 \;\Rightarrow\; s(t) = -18(t-22) + C$. But $s(22) = 424 = C \;\Rightarrow\; s(t) = -18(t-22) + 424$.

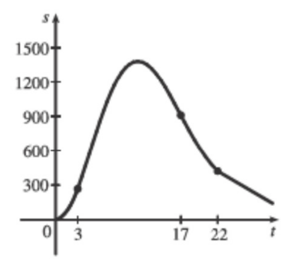
Therefore, until the rocket lands, we have

$$v(t) = \begin{cases} 30t^2 & \text{if } 0 \le t \le 3 \\ -32\,(t-3) + 270 & \text{if } 3 < t \le 17 \\ 32(t-17) - 178 & \text{if } 17 < t \le 22 \\ -18 & \text{if } t > 22 \end{cases}$$

and

$$s(t) = \begin{cases} 10t^3 & \text{if } 0 \le t \le 3 \\ -16(t-3)^2 + 270(t-3) + 270 & \text{if } 3 < t \le 17 \\ 16(t-17)^2 - 178\,(t-17) + 914 & \text{if } 17 < t \le 22 \\ -18(t-22) + 424 & \text{if } t > 22 \end{cases}$$





(b) To find the maximum height, set $v(t)$ on $3 < t \le 17$ equal to 0. $-32(t-3) + 270 = 0 \;\Rightarrow\; t_1 = 11.4375$ s and the maximum height is $s(t_1) = -16(t_1 - 3)^2 + 270(t_1 - 3) + 270 = 1409.0625$ ft.

(c) To find the time to land, set $s(t) = -18(t-22) + 424 = 0$. Then $t - 22 = \frac{424}{18} = 23.\overline{5}$, so $t \approx 45.6$ s.