3.1

4. (a) (i) Using Definition 1 with $f(x) = x - x^3$ and $P(1, 0)$,

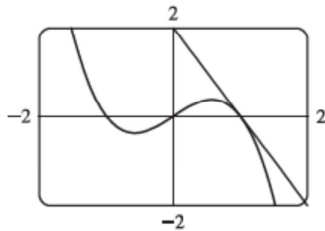$$m = \lim_{x \to 1} \frac{f(x) - 0}{x - 1} = \lim_{x \to 1} \frac{x - x^3}{x - 1} = \lim_{x \to 1} \frac{x(1 - x^2)}{x - 1} = \lim_{x \to 1} \frac{x(1 + x)(1 - x)}{x - 1}$$

$$= \lim_{x \to 1} \left[-x(1 + x) \right] = -1(2) = -2$$

(ii) Using Equation 2 with $f(x) = x - x^3$ and $P(1, 0)$,

$$m = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h} = \lim_{h \to 0} \frac{f(1 + h) - f(1)}{h} = \lim_{h \to 0} \frac{\left[(1 + h) - (1 + h)^3\right] - 0}{h}$$

$$= \lim_{h \to 0} \frac{1 + h - (1 + 3h + 3h^2 + h^3)}{h} = \lim_{h \to 0} \frac{-h^3 - 3h^2 - 2h}{h} = \lim_{h \to 0} \frac{h(-h^2 - 3h - 2)}{h}$$

$$= \lim_{h \to 0} (-h^2 - 3h - 2) = -2$$

(b) An equation of the tangent line is $y - f(a) = f'(a)(x - a) \Rightarrow y - f(1) = f'(1)(x - 1) \Rightarrow y - 0 = -2(x - 1)$,

or $y = -2x + 2$.

(c)

The graph of $y = -2x + 2$ is tangent to the graph of $y = x - x^3$ at the point $(1, 0)$. Now zoom in toward the point $(1, 0)$ until the cubic and the tangent line are indistinguishable.

12. (a) **Runner A** runs the entire 100-meter race at the same velocity since the slope of the position function is constant. **Runner B** starts the race at a slower velocity than runner A, but finishes the race at a faster velocity.

(b) The distance between the runners is the greatest at the time when the largest vertical line segment fits between the two graphs—this appears to be somewhere between 9 and 10 seconds.

(c) The runners had the same velocity when the slopes of their respective position functions are equal—this also appears to be at about 9.5 s. Note that the answers for parts (b) and (c) must be the same for these graphs because as soon as the velocity for runner B overtakes the velocity for runner A, the distance between the runners starts to decrease.

20. We begin by drawing a curve through the origin with a slope of 0 to satisfy $g(0) = 0$ and $g'(0) = 0$. The curve should have a slope of $-1, 3$, and 1 as we pass over $x = -1, 1$, and 2, respectively.
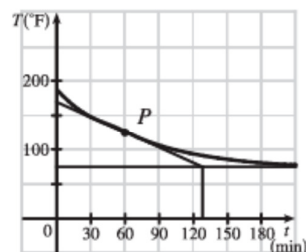
Note: In the figure, $y' = 0$ when $x \approx -1.27$ or 2.13.
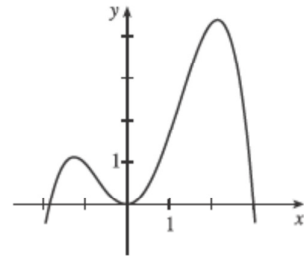
26. $f'(a) = \lim_{h \to 0} \dfrac{f(a + h) - f(a)}{h} = \lim_{h \to 0} \dfrac{\left[(a + h)^4 - 5(a + h)\right] - (a^4 - 5a)}{h}$

$$= \lim_{h \to 0} \frac{(a^4 + 4a^3 h + 6a^2 h^2 + 4ah^3 + h^4 - 5a - 5h) - (a^4 - 5a)}{h} = \lim_{h \to 0} \frac{4a^3 h + 6a^2 h^2 + 4ah^3 + h^4 - 5h}{h}$$

$$= \lim_{h \to 0} \frac{h(4a^3 + 6a^2 h + 4ah^2 + h^3 - 5)}{h} = \lim_{h \to 0} (4a^3 + 6a^2 h + 4ah^2 + h^3 - 5) = 4a^3 - 5$$

40. The slope of the tangent (that is, the rate of change of temperature with respect to time) at $t = 1$ h seems to be about $\dfrac{75 - 168}{132 - 0} \approx -0.7\,°\text{F/min}$.

42. (a) (i) $[2000, 2002]$: $\dfrac{P(2002) - P(2000)}{2002 - 2000} = \dfrac{5886 - 3501}{2} = \dfrac{2385}{2} = 1192.5$ locations/year

(ii) $[2000, 2001]$: $\dfrac{P(2001) - P(2000)}{2001 - 2000} = \dfrac{4709 - 3501}{1} = 1208$ locations/year
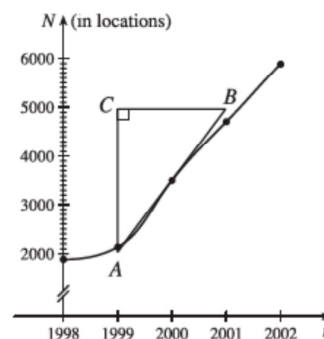
(iii) $[1999, 2000]$: $\dfrac{P(2000) - P(1999)}{2000 - 1999} = \dfrac{3501 - 2135}{1} = 1366$ locations/year

(b) Using the values from (ii) and (iii), we have

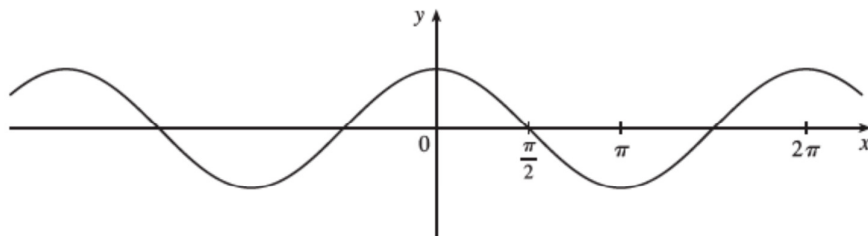$$\dfrac{1208 + 1366}{2} = 1287 \text{ locations/year.}$$

(c) Estimating A as $(1999, 2035)$ and B as $(2001, 4960)$, the slope at 2000 is

$$\dfrac{4960 - 2035}{2001 - 1999} = \dfrac{2925}{2} = 1462.5 \text{ locations/year.}$$

3.2

14.

The graph of the derivative looks like the graph of the cosine function.

40. Where d has horizontal tangents, only c is 0, so $d' = c$. c has negative tangents for $x < 0$ and b is the only graph that is negative for $x < 0$, so $c' = b$. b has positive tangents on $\mathbb{R}$ (except at $x = 0$), and the only graph that is positive on the same domain is a, so $b' = a$. We conclude that $d = f$, $c = f'$, $b = f''$, and $a = f'''$.

32. (a) $P'(t)$ is the rate at which the percentage of Americans under the age of 18 is changing with respect to time. Its units are percent per year (%/yr).

(b) To find $P'(t)$, we use $\displaystyle\lim_{h \to 0} \frac{P(t+h) - P(t)}{h} \approx \frac{P(t+h) - P(t)}{h}$ for small values of h.

For 1950: $P'(1950) \approx \dfrac{P(1960) - P(1950)}{1960 - 1950} = \dfrac{35.7 - 31.1}{10} = 0.46$

For 1960: We estimate $P'(1960)$ by using $h = -10$ and $h = 10$, and then average the two results to obtain a final estimate.
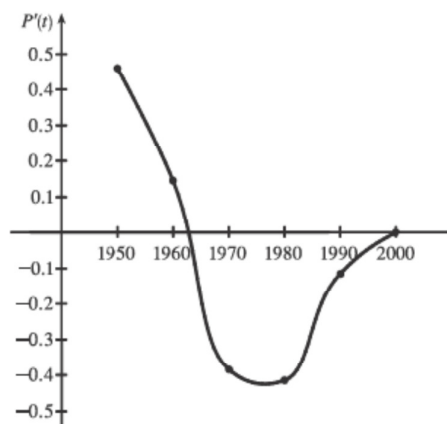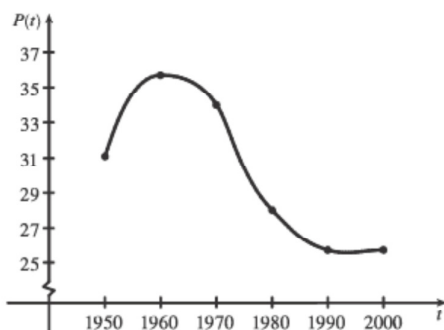
$h = -10 \quad\Rightarrow\quad P'(1960) \approx \dfrac{P(1950) - P(1960)}{1950 - 1960} = \dfrac{31.1 - 35.7}{-10} = 0.46$

$h = 10 \quad\Rightarrow\quad P'(1960) \approx \dfrac{P(1970) - P(1960)}{1970 - 1960} = \dfrac{34.0 - 35.7}{10} = -0.17$

So we estimate that $P'(1960) \approx \frac{1}{2}[0.46 + (-0.17)] = 0.145$.

t	1950	1960	1970	1980	1990	2000
$P'(t)$	0.460	0.145	−0.385	−0.415	−0.115	0.000

(c)

(d) We could get more accurate values for $P'(t)$ by obtaining data for the mid-decade years 1955, 1965, 1975, 1985, and 1995.

54. (a)

(b) The initial temperature of the water is close to room temperature because of the water that was in the pipes. When the water from the hot water tank starts coming out, dT/dt is large and positive as T increases to the temperature of the water in the tank. In the next phase, $dT/dt = 0$ as the water comes out at a constant, high temperature. After some time, dT/dt becomes small and negative as the contents of the hot water tank are exhausted. Finally, when the hot water has run out, dT/dt is once again 0 as the water maintains its (cold) temperature.

(c)

3.3

24. $Y(u) = (u^{-2} + u^{-3})(u^5 - 2u^2)$ $\overset{PR}{\Rightarrow}$

$$Y'(u) = (u^{-2} + u^{-3})(5u^4 - 4u) + (u^5 - 2u^2)(-2u^{-3} - 3u^{-4})$$

$$= (5u^2 - 4u^{-1} + 5u - 4u^{-2}) + (-2u^2 - 3u + 4u^{-1} + 6u^{-2}) = 3u^2 + 2u + 2u^{-2}$$

54. $y = (1 + 2x)^2 = 1 + 4x + 4x^2$ $\Rightarrow$ $y' = 4 + 8x$. At $(1, 9)$, $y' = 12$ and an equation of the tangent line is

$y - 9 = 12(x - 1)$ or $y = 12x - 3$. The slope of the normal line is $-\frac{1}{12}$ (the negative reciprocal of 12) and an equation of the

normal line is $y - 9 = -\frac{1}{12}(x - 1)$ or $y = -\frac{1}{12}x + \frac{109}{12}$.

86. $y = ax^2 + bx + c$ $\Rightarrow$ $y'(x) = 2ax + b$. The parabola has slope 4 at $x = 1$ and slope -8 at $x = -1$, so $y'(1) = 4$ $\Rightarrow$

$2a + b = 4$ (1) and $y'(-1) = -8$ $\Rightarrow$ $-2a + b = -8$ (2). Adding (1) and (2) gives us $2b = -4$ $\Leftrightarrow$ $b = -2$. From

(1), $2a - 2 = 4$ $\Leftrightarrow$ $a = 3$. Thus, the equation of the parabola is $y = 3x^2 - 2x + c$. Since it passes through the point

$(2, 15)$, we have $15 = 3(2)^2 - 2(2) + c$ $\Rightarrow$ $c = 7$, so the equation is $y = 3x^2 - 2x + 7$.
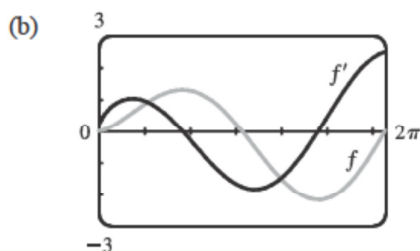
3.4

12. $y = \dfrac{1 - \sec x}{\tan x}$ $\Rightarrow$

$$y' = \frac{\tan x \, (-\sec x \tan x) - (1 - \sec x)(\sec^2 x)}{(\tan x)^2} = \frac{\sec x \, (-\tan^2 x - \sec x + \sec^2 x)}{\tan^2 x} = \frac{\sec x \, (1 - \sec x)}{\tan^2 x}$$

20. $f(x) = \cos x$ $\Rightarrow$

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} = \lim_{h \to 0} \frac{\cos(x + h) - \cos x}{h} = \lim_{h \to 0} \frac{\cos x \cos h - \sin x \sin h - \cos x}{h}$$

$$= \lim_{h \to 0} \left(\cos x \, \frac{\cos h - 1}{h} - \sin x \, \frac{\sin h}{h} \right) = \cos x \lim_{h \to 0} \frac{\cos h - 1}{h} - \sin x \lim_{h \to 0} \frac{\sin h}{h}$$

$$= (\cos x)(0) - (\sin x)(1) = -\sin x$$

28. (a) $f(x) = \sqrt{x} \sin x$ $\Rightarrow$ $f'(x) = \sqrt{x} \cos x + (\sin x)\left(\frac{1}{2}x^{-1/2}\right) = \sqrt{x} \cos x + \dfrac{\sin x}{2\sqrt{x}}$

(b)

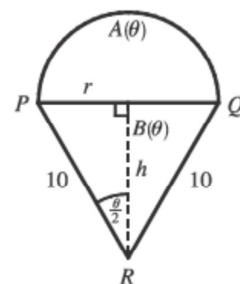Notice that $f'(x) = 0$ when f has a horizontal tangent.

f' is positive when f is increasing and f' is negative when f is decreasing.

50. We get the following formulas for r and h in terms of θ:

$$\sin \frac{\theta}{2} = \frac{r}{10} \quad \Rightarrow \quad r = 10 \sin \frac{\theta}{2} \quad \text{and} \quad \cos \frac{\theta}{2} = \frac{h}{10} \quad \Rightarrow \quad h = 10 \cos \frac{\theta}{2}$$

Now $A(\theta) = \frac{1}{2}\pi r^2$ and $B(\theta) = \frac{1}{2}(2r)h = rh$. So

$$\lim_{\theta \to 0^+} \frac{A(\theta)}{B(\theta)} = \lim_{\theta \to 0^+} \frac{\frac{1}{2}\pi r^2}{rh} = \frac{1}{2}\pi \lim_{\theta \to 0^+} \frac{r}{h} = \frac{1}{2}\pi \lim_{\theta \to 0^+} \frac{10 \sin(\theta/2)}{10 \cos(\theta/2)}$$

$$= \frac{1}{2}\pi \lim_{\theta \to 0^+} \tan(\theta/2) = 0$$

3.5

4. Let $u = g(x) = \sin x$ and $y = f(u) = \tan u$. Then $\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx} = (\sec^2 u)(\cos x) = \sec^2(\sin x) \cdot \cos x$,

or equivalently, $[\sec(\sin x)]^2 \cos x$.

34. $y = x \sin \dfrac{1}{x} \;\Rightarrow\; y' = \sin\dfrac{1}{x} + x \cos\dfrac{1}{x}\left(-\dfrac{1}{x^2}\right) = \sin\dfrac{1}{x} - \dfrac{1}{x}\cos\dfrac{1}{x}$

3.6

2. (a) $\dfrac{d}{dx}(4x^2 + 9y^2) = \dfrac{d}{dx}(36) \;\Rightarrow\; 8x + 18y \cdot y' = 0 \;\Rightarrow\; y' = -\dfrac{8x}{18y} = -\dfrac{4x}{9y}$

(b) $4x^2 + 9y^2 = 36 \;\Rightarrow\; 9y^2 = 36 - 4x^2 \;\Rightarrow\; y^2 = \tfrac{4}{9}(9 - x^2) \;\Rightarrow\; y = \pm\tfrac{2}{3}\sqrt{9 - x^2}$, so

$y' = \pm\tfrac{2}{3} \cdot \tfrac{1}{2}(9 - x^2)^{-1/2}(-2x) = \mp\dfrac{2x}{3\sqrt{9 - x^2}}$

(c) From part (a), $y' = -\dfrac{4x}{9y} = -\dfrac{4x}{9\left(\pm\tfrac{2}{3}\sqrt{9 - x^2}\right)} = \mp\dfrac{2x}{3\sqrt{9 - x^2}}$.

20. $\sin x + \cos y = \sin x \cos y \;\Rightarrow\; \cos x - \sin y \cdot y' = \sin x \,(-\sin y \cdot y') + \cos y \cos x \;\Rightarrow\;$

$(\sin x \sin y - \sin y)\, y' = \cos x \cos y - \cos x \;\Rightarrow\; y' = \dfrac{\cos x\,(\cos y - 1)}{\sin y\,(\sin x - 1)}$

46. The circles $x^2 + y^2 = ax$ and $x^2 + y^2 = by$ intersect at the origin where the tangents are vertical and horizontal [assume a and b are both nonzero]. If (x_0, y_0) is the other point of intersection, then $x_0^2 + y_0^2 = ax_0$ (1) and $x_0^2 + y_0^2 = by_0$ (2).

Now $x^2 + y^2 = ax \;\Rightarrow\; 2x + 2yy' = a \;\Rightarrow\; y' = \dfrac{a - 2x}{2y}$ and $x^2 + y^2 = by \;\Rightarrow$

$2x + 2yy' = by' \;\Rightarrow\; y' = \dfrac{2x}{b - 2y}$. Thus, the curves are orthogonal at $(x_0, y_0) \;\Leftrightarrow$

$\dfrac{a - 2x_0}{2y_0} = -\dfrac{b - 2y_0}{2x_0} \;\Leftrightarrow\; 2ax_0 - 4x_0^2 = 4y_0^2 - 2by_0 \;\Leftrightarrow\; ax_0 + by_0 = 2(x_0^2 + y_0^2)$,

which is true by (1) and (2).
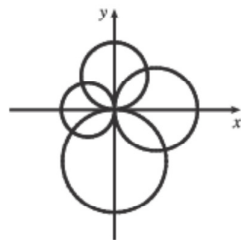
3.7

6. (a) The velocity v is positive when s is increasing, that is, on the intervals $(0, 1)$ and $(3, 4)$; and it is negative when s is decreasing, that is, on the interval $(1, 3)$. The acceleration a is positive when the graph of s is concave upward (v is increasing), that is, on the interval $(2, 4)$; and it is negative when the graph of s is concave downward (v is decreasing), that is, on the interval $(0, 2)$. The particle is speeding up on the interval $(1, 2)$ $[v < 0, a < 0]$ and on $(3, 4)$ $[v > 0, a > 0]$. The particle is slowing down on the interval $(0, 1)$ $[v > 0, a < 0]$ and on $(2, 3)$ $[v < 0, a > 0]$.

(b) The velocity v is positive on $(3, 4)$ and negative on $(0, 3)$. The acceleration a is positive on $(0, 1)$ and $(2, 4)$ and negative on $(1, 2)$. The particle is speeding up on the interval $(1, 2)$ $[v < 0, a < 0]$ and on $(3, 4)$ $[v > 0, a > 0]$. The particle is slowing down on the interval $(0, 1)$ $[v < 0, a > 0]$ and on $(2, 3)$ $[v < 0, a > 0]$.

12. (a) $V(x) = x^3 \Rightarrow \dfrac{dV}{dx} = 3x^2$. $\dfrac{dV}{dx}\bigg|_{x=3} = 3(3)^2 = 27$ mm^3/mm is the

rate at which the volume is increasing as x increases past 3 mm.
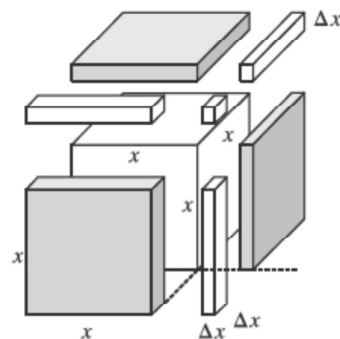
(b) The surface area is $S(x) = 6x^2$, so $V'(x) = 3x^2 = \frac{1}{2}(6x^2) = \frac{1}{2}S(x)$.

The figure suggests that if Δx is small, then the change in the volume of the

cube is approximately half of its surface area (the area of 3 of the 6 faces)

times Δx. From the figure, $\Delta V = 3x^2(\Delta x) + 3x(\Delta x)^2 + (\Delta x)^3$.

If Δx is small, then $\Delta V \approx 3x^2(\Delta x)$ and so $\Delta V/\Delta x \approx 3x^2$.

18. $V(t) = 5000\left(1 - \frac{1}{40}t\right)^2 \Rightarrow V'(t) = 5000 \cdot 2\left(1 - \frac{1}{40}t\right)\left(-\frac{1}{40}\right) = -250\left(1 - \frac{1}{40}t\right)$

(a) $V'(5) = -250\left(1 - \frac{5}{40}\right) = -218.75$ gal/min   (b) $V'(10) = -250\left(1 - \frac{10}{40}\right) = -187.5$ gal/min

(c) $V'(20) = -250\left(1 - \frac{20}{40}\right) = -125$ gal/min   (d) $V'(40) = -250\left(1 - \frac{40}{40}\right) = 0$ gal/min

The water is flowing out the fastest at the beginning—when $t = 0$, $V'(t) = -250$ gal/min. The water is flowing out the

slowest at the end—when $t = 40$, $V'(t) = 0$. As the tank empties, the water flows out more slowly.

26. (a) (i) $f = \dfrac{1}{2L}\sqrt{\dfrac{T}{\rho}} = \left(\dfrac{1}{2}\sqrt{\dfrac{T}{\rho}}\right)L^{-1} \Rightarrow \dfrac{df}{dL} = -\left(\dfrac{1}{2}\sqrt{\dfrac{T}{\rho}}\right)L^{-2} = -\dfrac{1}{2L^2}\sqrt{\dfrac{T}{\rho}}$

(ii) $f = \dfrac{1}{2L}\sqrt{\dfrac{T}{\rho}} = \left(\dfrac{1}{2L\sqrt{\rho}}\right)T^{1/2} \Rightarrow \dfrac{df}{dT} = \dfrac{1}{2}\left(\dfrac{1}{2L\sqrt{\rho}}\right)T^{-1/2} = \dfrac{1}{4L\sqrt{T\rho}}$
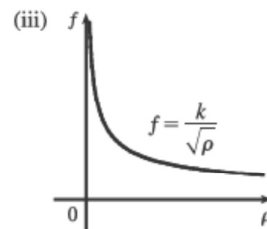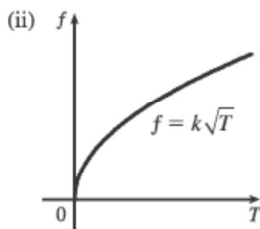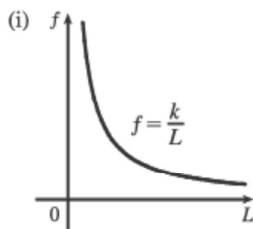
(iii) $f = \dfrac{1}{2L}\sqrt{\dfrac{T}{\rho}} = \left(\dfrac{\sqrt{T}}{2L}\right)\rho^{-1/2} \Rightarrow \dfrac{df}{d\rho} = -\dfrac{1}{2}\left(\dfrac{\sqrt{T}}{2L}\right)\rho^{-3/2} = -\dfrac{\sqrt{T}}{4L\rho^{3/2}}$

(b) *Note:* Illustrating tangent lines on the generic figures may help to explain the results.

(i) $\dfrac{df}{dL} < 0$ and L is decreasing $\Rightarrow$ f is increasing $\Rightarrow$ higher note

(ii) $\dfrac{df}{dT} > 0$ and T is increasing $\Rightarrow$ f is increasing $\Rightarrow$ higher note

(iii) $\dfrac{df}{d\rho} < 0$ and ρ is increasing $\Rightarrow$ f is decreasing $\Rightarrow$ lower note

3.8

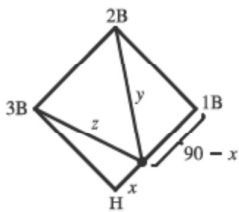2. (a) $A = \pi r^2 \Rightarrow \dfrac{dA}{dt} = \dfrac{dA}{dr}\dfrac{dr}{dt} = 2\pi r\dfrac{dr}{dt}$ (b) $\dfrac{dA}{dt} = 2\pi r\dfrac{dr}{dt} = 2\pi(30\text{ m})(1\text{ m/s}) = 60\pi$ m^2/s

8. $x^2 + y^2 = 25 \Rightarrow 2x\dfrac{dx}{dt} + 2y\dfrac{dy}{dt} = 0 \Rightarrow x\dfrac{dx}{dt} = -y\dfrac{dy}{dt} \Rightarrow \dfrac{dx}{dt} = -\dfrac{y}{x}\dfrac{dy}{dt}$.

When $y = 4$, $x^2 + 4^2 = 25 \Rightarrow x = \pm 3$. For $\dfrac{dy}{dt} = 6$, $\dfrac{dx}{dt} = -\dfrac{4}{\pm 3}(6) = \mp 8$.

18. We are given that $\dfrac{dx}{dt} = 24$ ft/s.

(a)

$y^2 = (90-x)^2 + 90^2 \;\Rightarrow\; 2y\dfrac{dy}{dt} = 2(90-x)\left(-\dfrac{dx}{dt}\right)$. When $x = 45$,

$y = \sqrt{45^2 + 90^2} = 45\sqrt{5}$, so $\dfrac{dy}{dt} = \dfrac{90-x}{y}\left(-\dfrac{dx}{dt}\right) = \dfrac{45}{45\sqrt{5}}(-24) = -\dfrac{24}{\sqrt{5}}$,

so the distance from second base is decreasing at a rate of $\dfrac{24}{\sqrt{5}} \approx 10.7$ ft/s.

(b) Due to the symmetric nature of the problem in part (a), we expect to get the same answer—and we do.

$z^2 = x^2 + 90^2 \;\Rightarrow\; 2z\dfrac{dz}{dt} = 2x\dfrac{dx}{dt}$. When $x = 45$, $z = 45\sqrt{5}$, so $\dfrac{dz}{dt} = \dfrac{45}{45\sqrt{5}}(24) = \dfrac{24}{\sqrt{5}} \approx 10.7$ ft/s.

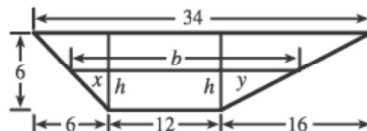**26.** The figure is drawn without the top 3 feet.

$V = \frac{1}{2}(b+12)h(20) = 10(b+12)h$ and, from similar triangles,

$\dfrac{x}{h} = \dfrac{6}{6}$ and $\dfrac{y}{h} = \dfrac{16}{6} = \dfrac{8}{3}$, so $b = x + 12 + y = h + 12 + \dfrac{8h}{3} = 12 + \dfrac{11h}{3}$.

Thus, $V = 10\left(24 + \dfrac{11h}{3}\right)h = 240h + \dfrac{110h^2}{3}$ and so $0.8 = \dfrac{dV}{dt} = \left(240 + \dfrac{220}{3}h\right)\dfrac{dh}{dt}$.
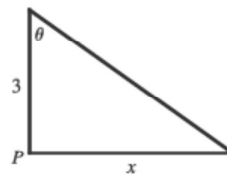
When $h = 5$, $\dfrac{dh}{dt} = \dfrac{0.8}{240 + 5(220/3)} = \dfrac{3}{2275} \approx 0.00132$ ft/min.

38. We are given that $\dfrac{d\theta}{dt} = 4(2\pi) = 8\pi$ rad/min. $x = 3\tan\theta \;\Rightarrow$

$\dfrac{dx}{dt} = 3\sec^2\theta\,\dfrac{d\theta}{dt}$. When $x = 1$, $\tan\theta = \frac{1}{3}$, so $\sec^2\theta = 1 + \left(\frac{1}{3}\right)^2 = \dfrac{10}{9}$

and $\dfrac{dx}{dt} = 3\left(\dfrac{10}{9}\right)(8\pi) = \dfrac{80}{3}\pi \approx 83.8$ km/min.

44. The hour hand of a clock goes around once every 12 hours or, in radians per hour,

$\dfrac{2\pi}{12} = \dfrac{\pi}{6}$ rad/h. The minute hand goes around once an hour, or at the rate of 2π rad/h.

So the angle θ between them (measuring clockwise from the minute hand to the hour

hand) is changing at the rate of $d\theta/dt = \dfrac{\pi}{6} - 2\pi = -\dfrac{11\pi}{6}$ rad/h. Now, to relate θ to ℓ,

we use the Law of Cosines: $\ell^2 = 4^2 + 8^2 - 2\cdot4\cdot8\cdot\cos\theta = 80 - 64\cos\theta$ $(\star)$.

Differentiating implicitly with respect to t, we get $2\ell\,\dfrac{d\ell}{dt} = -64(-\sin\theta)\dfrac{d\theta}{dt}$. At 1:00, the angle between the two hands is

one-twelfth of the circle, that is, $\dfrac{2\pi}{12} = \dfrac{\pi}{6}$ radians. We use $(\star)$ to find ℓ at 1:00: $\ell = \sqrt{80 - 64\cos\frac{\pi}{6}} = \sqrt{80 - 32\sqrt{3}}$.

Substituting, we get $2\ell\,\dfrac{d\ell}{dt} = 64\sin\dfrac{\pi}{6}\left(-\dfrac{11\pi}{6}\right) \;\Rightarrow\; \dfrac{d\ell}{dt} = \dfrac{64\left(\frac{1}{2}\right)\left(-\frac{11\pi}{6}\right)}{2\sqrt{80 - 32\sqrt{3}}} = -\dfrac{88\pi}{3\sqrt{80 - 32\sqrt{3}}} \approx -18.6$.

So at 1:00, the distance between the tips of the hands is decreasing at a rate of 18.6 mm/h ≈ 0.005 mm/s.
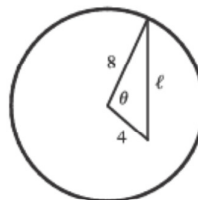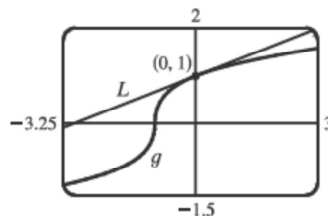
3.9

6. $g(x) = \sqrt[3]{1+x} = (1+x)^{1/3} \;\Rightarrow\; g'(x) = \frac{1}{3}(1+x)^{-2/3}$, so $g(0) = 1$ and

$g'(0) = \frac{1}{3}$. Therefore, $\sqrt[3]{1+x} = g(x) \approx g(0) + g'(0)(x-0) = 1 + \frac{1}{3}x$.

So $\sqrt[3]{0.95} = \sqrt[3]{1+(-0.05)} \approx 1 + \frac{1}{3}(-0.05) = 0.98\overline{3}$,

and $\sqrt[3]{1.1} = \sqrt[3]{1+0.1} \approx 1 + \frac{1}{3}(0.1) = 1.0\overline{3}$.

12. (a) For $y = f(s) = \dfrac{s}{1+2s}$, $f'(s) = \dfrac{(1+2s)(1) - s(2)}{(1+2s)^2} = \dfrac{1}{(1+2s)^2}$, so $dy = \dfrac{1}{(1+2s)^2}\,ds$.

(b) $y = u\cos u \;\Rightarrow\; y' = u(-\sin u) + \cos u \cdot 1 \;\Rightarrow\; dy = (\cos u - u\sin u)\,du$

20. $y = f(x) = \sqrt{x}$, $x = 1$, $\Delta x = 1 \;\Rightarrow$

$\Delta y = f(2) - f(1) = \sqrt{2} - \sqrt{1} = \sqrt{2} - 1 \approx 0.414$

$dy = \dfrac{1}{2\sqrt{x}}\,dx = \tfrac{1}{2}(1) = 0.5$