2.1

2. (a) Slope $= \frac{2948-2530}{42-36} = \frac{418}{6} \approx 69.67$

(b) Slope $= \frac{2948-2661}{42-38} = \frac{287}{4} = 71.75$

(c) Slope $= \frac{2948-2806}{42-40} = \frac{142}{2} = 71$

(d) Slope $= \frac{3080-2948}{44-42} = \frac{132}{2} = 66$

From the data, we see that the patient's heart rate is decreasing from 71 to 66 heartbeats/minute after 42 minutes. After being stable for a while, the patient's heart rate is dropping.

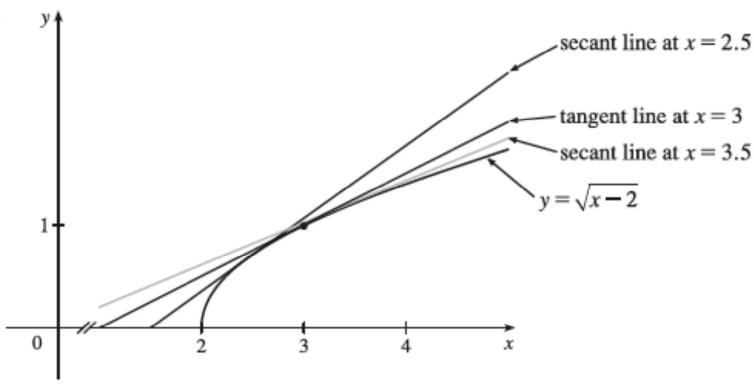
4. (a)

	x	Q	m_{PQ}
(i)	2.5	(2.5, 0.707107)	0.585786
(ii)	2.9	(2.9, 0.948683)	0.513167
(iii)	2.99	(2.99, 0.994987)	0.501256
(iv)	2.999	(2.999, 0.999500)	0.500125
(v)	3.5	(3.5, 1.224745)	0.449490
(vi)	3.1	(3.1, 1.048809)	0.488088
(vii)	3.01	(3.01, 1.004988)	0.498756
(viii)	3.001	(3.001, 1.000500)	0.499875

(b) The slope appears to be $\frac{1}{2}$.

(c) $y - 1 = \frac{1}{2}(x-3)$ or $y = \frac{1}{2}x - \frac{1}{2}$.

(d)



2.2

10. $\lim_{t\to 12^-} f(t) = 150$ mg and $\lim_{t\to 12^+} f(t) = 300$ mg. These limits show that there is an abrupt change in the amount of drug in the patient's bloodstream at $t = 12$ h. The left-hand limit represents the amount of the drug just before the fourth injection. The right-hand limit represents the amount of the drug just after the fourth injection.

22. For $f(x) = \dfrac{\tan 3x}{\tan 5x}$:

x	$f(x)$
± 0.2	0.439279
± 0.1	0.566236
± 0.05	0.591893
± 0.01	0.599680
± 0.001	0.599997

It appears that $\displaystyle\lim_{x\to 0} \frac{\tan 3x}{\tan 5x} = 0.6 = \frac{3}{5}$.

40. $\lim_{v \to c^-} m = \lim_{v \to c^-} \dfrac{m_0}{\sqrt{1 - v^2/c^2}}$. As $v \to c^-$, $\sqrt{1 - v^2/c^2} \to 0^+$, and $m \to \infty$.

2.3

6.
$$\begin{aligned}\lim_{t \to -1} (t^2+1)^3(t+3)^5 &= \lim_{t \to -1} (t^2+1)^3 \cdot \lim_{t \to -1} (t+3)^5 && \text{[Limit Law 4]}\\ &= \left[\lim_{t \to -1} (t^2+1)\right]^3 \cdot \left[\lim_{t \to -1} (t+3)\right]^5 && \text{[6]}\\ &= \left[\lim_{t \to -1} t^2 + \lim_{t \to -1} 1\right]^3 \cdot \left[\lim_{t \to -1} t + \lim_{t \to -1} 3\right]^5 && \text{[1]}\\ &= \left[(-1)^2+1\right]^3 \cdot [-1+3]^5 = 8 \cdot 32 = 256 && \text{[9, 7, and 8]}\end{aligned}$$

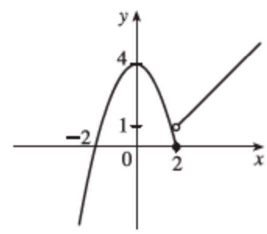
18. $\lim_{x \to 1} \dfrac{x^3-1}{x^2-1} = \lim_{x \to 1} \dfrac{(x-1)(x^2+x+1)}{(x-1)(x+1)} = \lim_{x \to 1} \dfrac{x^2+x+1}{x+1} = \dfrac{1^2+1+1}{1+1} = \dfrac{3}{2}$

46. (a) $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (4 - x^2) = \lim_{x \to 2^-} 4 - \lim_{x \to 2^-} x^2 = 4 - 4 = 0$

$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (x-1) = \lim_{x \to 2^+} x - \lim_{x \to 2^+} 1 = 2 - 1 = 1$

(b) No, $\lim_{x \to 2} f(x)$ does not exist since $\lim_{x \to 2^-} f(x) \neq \lim_{x \to 2^+} f(x)$.

(c)

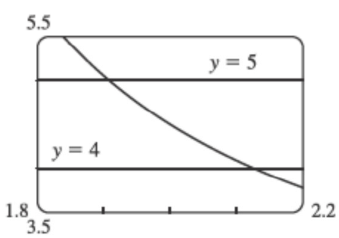


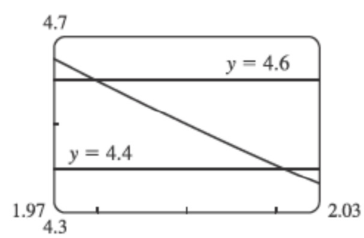
58. Let $f(x) = [\![x]\!]$ and $g(x) = -[\![x]\!]$. Then $\lim_{x \to 3} f(x)$ and $\lim_{x \to 3} g(x)$ do not exist [Example 10]

but $\lim_{x \to 3} [f(x) + g(x)] = \lim_{x \to 3} ([\![x]\!] - [\![x]\!]) = \lim_{x \to 3} 0 = 0$.

2.4

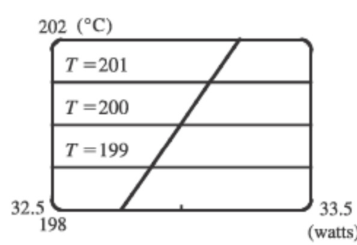
8. For $y = (4x+1)/(3x-4)$ and $\varepsilon = 0.5$, we need $1.91 \le x \le 2.125$. So since $|2 - 1.91| = 0.09$ and $|2 - 2.125| = 0.125$, we can take $0 < \delta \le 0.09$. For $\varepsilon = 0.1$, we need $1.980 \le 2.021$. So since $|2 - 1.980| = 0.02$ and $|2 - 2.021| = 0.021$, we can take $\delta = 0.02$ (or any smaller positive number).





12. (a) $T = 0.1w^2 + 2.155w + 20$ and $T = 200 \Rightarrow$

$0.1w^2 + 2.155w + 20 = 200 \Rightarrow$ [by the quadratic formula or from the graph] $w \approx 33.0$ watts $(w > 0)$



(b) From the graph, $199 \le T \le 201 \Rightarrow 32.89 < w < 33.11$.

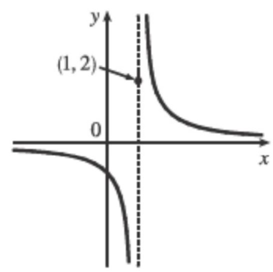
(c) x is the input power, $f(x)$ is the temperature, a is the target input power given in part (a), L is the target temperature (200), ε is the tolerance in the temperature (1), and δ is the tolerance in the power input in watts indicated in part (b) (0.11 watts).

2.5

10. $\lim_{x\to 4} f(x) = \lim_{x\to 4}\left(x^2 + \sqrt{7-x}\right) = \lim_{x\to 4} x^2 + \sqrt{\lim_{x\to 4} 7 - \lim_{x\to 4} x} = 4^2 + \sqrt{7-4} = 16 + \sqrt{3} = f(4)$.

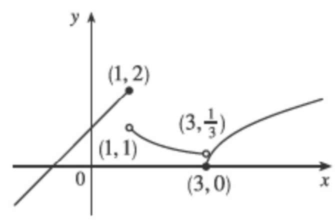
By the definition of continuity, f is continuous at $a = 4$.

16. $f(x) = \begin{cases} 1/(x-1) & \text{if } x \neq 1 \\ 2 & \text{if } x = 1 \end{cases}$ is discontinuous at 1 because $\lim_{x\to 1} f(x)$ does not exist.



32. Because x is continuous on $\mathbb{R}$, $\sin x$ is continuous on $\mathbb{R}$, and $x + \sin x$ is continuous on $\mathbb{R}$, the composite function $f(x) = \sin(x + \sin x)$ is continuous on $\mathbb{R}$, so $\lim_{x\to\pi} f(x) = f(\pi) = \sin(\pi + \sin\pi) = \sin\pi = 0$.

38. $f(x) = \begin{cases} x+1 & \text{if } x \leq 1 \\ 1/x & \text{if } 1 < x < 3 \\ \sqrt{x-3} & \text{if } x \geq 3 \end{cases}$



f is continuous on $(-\infty, 1)$, $(1, 3)$, and $(3, \infty)$, where it is a polynomial, a rational function, and a composite of a root function with a polynomial, respectively. Now $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-}(x+1) = 2$ and $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+}(1/x) = 1$, so f is discontinuous at 1. Since $f(1) = 2$, f is continuous from the left at 1. Also, $\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-}(1/x) = 1/3$, and $\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+}\sqrt{x-3} = 0 = f(3)$, so f is discontinuous at 3, but it is continuous from the right at 3.