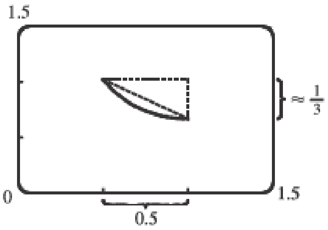
9.1

8. $y^2 = 4(x+4)^3,\ y > 0 \ \Rightarrow\ y = 2(x+4)^{3/2} \ \Rightarrow\ dy/dx = 3(x+4)^{1/2} \ \Rightarrow$

$1 + (dy/dx)^2 = 1 + 9(x+4) = 9x + 37$. So

$L = \int_0^2 \sqrt{9x+37}\,dx \quad \begin{bmatrix} u = 9x+37, \\ du = 9\,dx \end{bmatrix} \quad = \int_{37}^{55} u^{1/2}\left(\frac{1}{9}\,du\right) = \frac{1}{9}\cdot\frac{2}{3}\left[u^{3/2}\right]_{37}^{55} = \frac{2}{27}\left(55\sqrt{55} - 37\sqrt{37}\right).$

22. 

From the figure, the length of the curve is slightly larger than the hypotenuse of the triangle formed by the points $(0.5, f(0.5) \approx 1)$, $(1, f(0.5) \approx 1)$ and $(1, \frac{2}{3})$, where $y = f(x) = x^3/6 + 1/(2x)$. This length is about $\sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{3}\right)^2} \approx 0.6$, so we might estimate the length to be 0.65.

$$y = \frac{x^3}{6} + \frac{1}{2x} \ \Rightarrow\ y' = \frac{x^2}{2} - \frac{x^{-2}}{2} \ \Rightarrow\ 1 + (y')^2 = 1 + \frac{x^4}{4} - \frac{1}{2} + \frac{x^{-4}}{4} = \frac{x^4}{4} + \frac{1}{2} + \frac{x^{-4}}{4} = \left(\frac{x^2}{2} + \frac{x^{-2}}{2}\right)^2$$

so, using the fact that the parenthetical expression is positive,

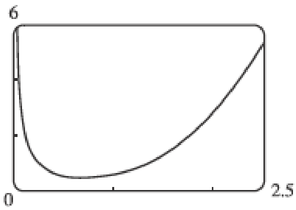
$$L = \int_{1/2}^{1} \sqrt{\left(\frac{x^2}{2} + \frac{x^{-2}}{2}\right)^2}\,dx = \int_{1/2}^{1}\left(\frac{x^2}{2} + \frac{x^{-2}}{2}\right)dx = \left[\frac{x^3}{6} - \frac{1}{2x}\right]_{1/2}^{1} = \left(\frac{1}{6} - \frac{1}{2}\right) - \left(\frac{1}{48} - 1\right) = \frac{31}{48} = 0.6458\overline{3}$$

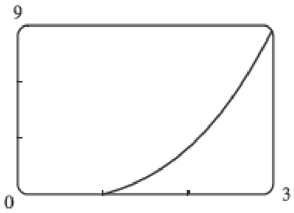
26. $y = x\ln x \ \Rightarrow\ dy/dx = 1 + \ln x$. Let $f(x) = \sqrt{1 + (dy/dx)^2} = \sqrt{1 + (1 + \ln x)^2}$. Then $L = \int_1^3 f(x)\,dx$.

Since $n = 10$, $\Delta x = \frac{3-1}{10} = \frac{1}{5}$. Now

$$L \approx S_{10} = \frac{1/5}{3}[f(1) + 4f(1.2) + 2f(1.4) + 4f(1.6) + 2f(1.8) + 4f(2)$$
$$+ 2f(2.2) + 4f(2.4) + 2f(2.6) + 4f(2.8) + f(3)] \approx 3.869618.$$

The value of the integral produced by a calculator is 3.869617 (to six decimal places).

34. (a) 

(c) 

(b) $f(x) = \frac{1}{3}x^3 + \frac{1}{4x},\ x > 0 \ \Rightarrow\ f'(x) = x^2 - \frac{1}{4x^2}$. Then

$$1 + [f'(x)]^2 = 1 + \left(x^4 - \frac{1}{2} + \frac{1}{16x^4}\right) = x^4 + \frac{1}{2} + \frac{1}{16x^4} = \left(x^2 + \frac{1}{4x^2}\right)^2,$$

so $\sqrt{1 + [f'(x)]^2} = x^2 + \frac{1}{4x^2}$. Thus,

$$s(x) = \int_1^x \sqrt{1 + [f'(t)]^2}\,dt = \int_1^x \left(t^2 + \frac{1}{4t^2}\right)dt = \left[\frac{1}{3}t^3 - \frac{1}{4t}\right]_1^x$$
$$= \left(\frac{1}{3}x^3 - \frac{1}{4x}\right) - \left(\frac{1}{3} - \frac{1}{4}\right) = \frac{1}{3}x^3 - \frac{1}{4x} - \frac{1}{12} \quad \text{for } x \ge 1$$

9.2

6. The curve $9x = y^2 + 18$ is symmetric about the x-axis, so we only use its top half, given by

$y = 3\sqrt{x-2}$. $\frac{dy}{dx} = \frac{3}{2\sqrt{x-2}}$, so $1 + \left(\frac{dy}{dx}\right)^2 = 1 + \frac{9}{4(x-2)}$. Thus,

$$S = \int_2^6 2\pi\cdot 3\sqrt{x-2}\sqrt{1 + \frac{9}{4(x-2)}}\,dx = 6\pi\int_2^6 \sqrt{x - 2 + \tfrac{9}{4}}\,dx = 6\pi\int_2^6 \left(x + \tfrac{1}{4}\right)^{1/2}dx$$
$$= 6\pi\cdot\tfrac{2}{3}\left[\left(x + \tfrac{1}{4}\right)^{3/2}\right]_2^6 = 4\pi\left[\left(\tfrac{25}{4}\right)^{3/2} - \left(\tfrac{9}{4}\right)^{3/2}\right] = 4\pi\left(\tfrac{125}{8} - \tfrac{27}{8}\right) = 4\pi\cdot\tfrac{98}{8} = 49\pi$$

28. In general, if the parabola $y = ax^2$, $-c \le x \le c$, is rotated about the y-axis, the surface area it generates is

$$2\pi \int_0^c x\sqrt{1+(2ax)^2}\,dx = 2\pi \int_0^{2ac} \frac{u}{2a}\sqrt{1+u^2}\,\frac{1}{2a}\,du \quad \begin{bmatrix} u = 2ax, \\ du = 2a\,dx \end{bmatrix} \quad = \frac{\pi}{4a^2}\int_0^{2ac}\left(1+u^2\right)^{1/2} 2u\,du$$

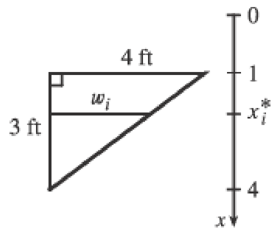
$$= \frac{\pi}{4a^2}\left[\tfrac{2}{3}\left(1+u^2\right)^{3/2}\right]_0^{2ac} = \frac{\pi}{6a^2}\left[\left(1+4a^2c^2\right)^{3/2}-1\right]$$

Here $2c = 10$ ft and $ac^2 = 2$ ft, so $c = 5$ and $a = \frac{2}{25}$. Thus, the surface area is

$$S = \frac{\pi}{6}\frac{625}{4}\left[\left(1+4\cdot\frac{4}{625}\cdot 25\right)^{3/2}-1\right] = \frac{625\pi}{24}\left[\left(1+\frac{16}{25}\right)^{3/2}-1\right] = \frac{625\pi}{24}\left(\frac{41\sqrt{41}}{125}-1\right) = \frac{5\pi}{24}\left(41\sqrt{41}-125\right) \approx 90.01\text{ ft}^2.$$

9.3

4. Set up a vertical x-axis as shown. Then the area of the ith rectangular strip is $\frac{4}{3}(4-x_i^*)\,\Delta x$. $\left[\text{By similar triangles, } \dfrac{w_i}{4-x_i^*} = \dfrac{4}{3}, \text{ so } w_i = \tfrac{4}{3}(4-x_i^*).\right]$



The pressure on the strip is δx_i^*, so the hydrostatic force on the strip is $\delta x_i^* \cdot \frac{4}{3}(4-x_i^*)\,\Delta x$ and the total force on the plate $\approx \sum_{i=1}^{n} \delta x_i^* \cdot \frac{4}{3}(4-x_i^*)\,\Delta x$. The total force

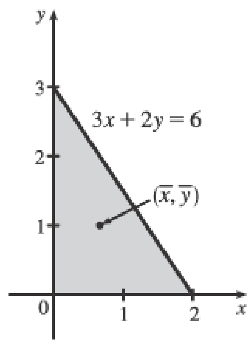
$$F = \lim_{n\to\infty}\sum_{i=1}^{n} \delta x_i^* \cdot \tfrac{4}{3}(4-x_i^*)\,\Delta x = \int_1^4 \delta x \cdot \tfrac{4}{3}(4-x)\,dx = \tfrac{4}{3}\delta\int_1^4 (4x - x^2)\,dx$$

$$= \tfrac{4}{3}\delta\left[2x^2 - \tfrac{1}{3}x^3\right]_1^4 = \tfrac{4}{3}\delta\left[\left(32-\tfrac{64}{3}\right)-\left(2-\tfrac{1}{3}\right)\right] = \tfrac{4}{3}\delta(9) = 12\delta \approx 750\text{ lb}$$

18. Partition the interval $[a,b]$ by points x_i as usual and choose $x_i^* \in [x_{i-1}, x_i]$ for each i. The ith horizontal strip of the immersed plate is approximated by a rectangle of height Δx_i and width $w(x_i^*)$, so its area is $A_i \approx w(x_i^*)\,\Delta x_i$. For small Δx_i, the pressure P_i on the ith strip is almost constant and $P_i \approx \rho g x_i^*$ by Equation 1. The hydrostatic force F_i acting on the ith strip is $F_i = P_i A_i \approx \rho g x_i^* w(x_i^*)\,\Delta x_i$. Adding these forces and taking the limit as $n \to \infty$, we obtain the hydrostatic force on the immersed plate:

$$F = \lim_{n\to\infty}\sum_{i=1}^{n} F_i = \lim_{n\to\infty}\sum_{i=1}^{n} \rho g x_i^* w(x_i^*)\,\Delta x_i = \int_a^b \rho g x w(x)\,dx$$

26. The region in the figure is "left-heavy" and "bottom-heavy," so we know $\overline{x} < 1$ and $\overline{y} < 1.5$, and we might guess that $\overline{x} = 0.7$ and $\overline{y} = 1.2$.



$3x + 2y = 6 \quad \Leftrightarrow \quad 2y = 6 - 3x \quad \Leftrightarrow \quad y = 3 - \frac{3}{2}x.$

$A = \int_0^2 \left(3 - \frac{3}{2}x\right)dx = \left[3x - \frac{3}{4}x^2\right]_0^2 = 6 - 3 = 3.$

$\overline{x} = \frac{1}{A}\int_0^2 x\left(3 - \frac{3}{2}x\right)dx = \frac{1}{3}\int_0^2 \left(3x - \frac{3}{2}x^2\right)dx = \frac{1}{3}\left[\frac{3}{2}x^2 - \frac{1}{2}x^3\right]_0^2$

$\quad = \frac{1}{3}(6-4) = \frac{2}{3}.$

$\overline{y} = \frac{1}{A}\int_0^2 \frac{1}{2}\left(3 - \frac{3}{2}x\right)^2 dx = \frac{1}{3}\cdot\frac{1}{2}\int_0^2 \left(9 - 9x + \frac{9}{4}x^2\right)dx = \frac{1}{6}\left[9x - \frac{9}{2}x^2 + \frac{3}{4}x^3\right]_0^2 = \frac{1}{6}(18 - 18 + 6) = 1.$

Thus, the centroid is $(\overline{x}, \overline{y}) = \left(\frac{2}{3}, 1\right)$.

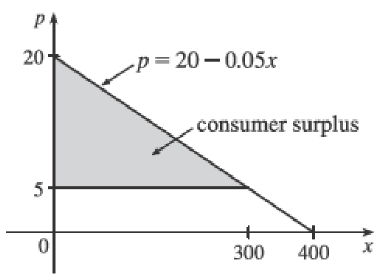
9.4

4. Consumer surplus $= \int_0^{300}\left[p(x) - p(300)\right]dx$

$\quad = \int_0^{300}[20 - 0.05x - (5)]\,dx$

$\quad = \int_0^{300}(15 - 0.05x)\,dx = \left[15x - 0.025x^2\right]_0^{300}$

$\quad = 4500 - 2250 = \$2250$



14. $n(9) - n(5) = \int_5^9 (2200 + 10e^{0.8t})\, dt = \left[2200t + \frac{10e^{0.8t}}{0.8}\right]_5^9 = [2200t]_5^9 + \frac{25}{2}\left[e^{0.8t}\right]_5^9$

$= 2200(9-5) + 12.5(e^{7.2} - e^4) \approx 24{,}860$

18. As in Example 2, we will estimate the cardiac output using Simpson's Rule with $\Delta t = (20-0)/10 = 2$.

$$\int_0^{20} c(t)\, dt \approx \tfrac{2}{3}[c(0) + 4c(2) + 2c(4) + 4c(6) + 2c(8) + 4c(10) + 2c(12) + 4c(14) + 2c(16) + 4c(18) + c(20)]$$
$$= \tfrac{2}{3}[0 + 4(2.4) + 2(5.1) + 4(7.8) + 2(7.6) + 4(5.4) + 2(3.9) + 4(2.3) + 2(1.6) + 4(0.7) + 0]$$
$$= \tfrac{2}{3}(110.8) \approx 73.87 \text{ mg}\cdot\text{s/L}$$

Therefore, $F \approx \dfrac{A}{73.87} = \dfrac{8}{73.87} \approx 0.1083$ L/s or 6.498 L/min.

9.5

2. (a) The probability that you drive to school in less than 15 minutes is $\int_0^{15} f(t)\, dt$.

(b) The probability that it takes you more than half an hour to get to school is $\int_{30}^{\infty} f(t)\, dt$.

8. (a) As in the preceding exercise, (1) $f(x) \geq 0$ and (2) $\int_{-\infty}^{\infty} f(x)\, dx = \int_0^{10} f(x)\, dx = \frac{1}{2}(10)(0.2)$ [area of a triangle] $= 1$.

So $f(x)$ is a probability density function.

(b) (i) $P(X < 3) = \int_0^3 f(x)\, dx = \frac{1}{2}(3)(0.1) = \frac{3}{20} = 0.15$

(ii) We first compute $P(X > 8)$ and then subtract that value and our answer in (i) from 1 (the total probability).

$P(X > 8) = \int_8^{10} f(x)\, dx = \frac{1}{2}(2)(0.1) = \frac{2}{20} = 0.10$. So $P(3 \leq X \leq 8) = 1 - 0.15 - 0.10 = 0.75$.

(c) We find equations of the lines from $(0, 0)$ to $(6, 0.2)$ and from $(6, 0.2)$ to $(10, 0)$, and find that

$$f(x) = \begin{cases} \frac{1}{30}x & \text{if } 0 \leq x < 6 \\ -\frac{1}{20}x + \frac{1}{2} & \text{if } 6 \leq x < 10 \\ 0 & \text{otherwise} \end{cases}$$

$$\mu = \int_{-\infty}^{\infty} x f(x)\, dx = \int_0^6 x\left(\tfrac{1}{30}x\right) dx + \int_6^{10} x\left(-\tfrac{1}{20}x + \tfrac{1}{2}\right) dx = \left[\tfrac{1}{90}x^3\right]_0^6 + \left[-\tfrac{1}{60}x^3 + \tfrac{1}{4}x^2\right]_6^{10}$$
$$= \tfrac{216}{90} + \left(-\tfrac{1000}{60} + \tfrac{100}{4}\right) - \left(-\tfrac{216}{60} + \tfrac{36}{4}\right) = \tfrac{16}{3} = 5.\overline{3}$$

10. (a) $\mu = 1000 \quad \Rightarrow \quad f(t) = \begin{cases} 0 & \text{if } t < 0 \\ \frac{1}{1000}e^{-t/1000} & \text{if } t \geq 0 \end{cases}$

(i) $P(0 \leq X \leq 200) = \int_0^{200} \frac{1}{1000}e^{-t/1000}\, dt = \left[-e^{-t/1000}\right]_0^{200} = -e^{-1/5} + 1 \approx 0.181$

(ii) $P(X > 800) = \int_{800}^{\infty} \frac{1}{1000}e^{-t/1000}\, dt = \lim_{x\to\infty}\left[-e^{-t/1000}\right]_{800}^{x} = 0 + e^{-4/5} \approx 0.449$

(b) We need to find m so that $\int_m^{\infty} f(t)\, dt = \frac{1}{2} \quad \Rightarrow \quad \lim_{x\to\infty} \int_m^x \frac{1}{1000}e^{-t/1000}\, dt = \frac{1}{2} \quad \Rightarrow \quad \lim_{x\to\infty}\left[-e^{-t/1000}\right]_m^x = \frac{1}{2} \quad \Rightarrow$

$0 + e^{-m/1000} = \frac{1}{2} \quad \Rightarrow \quad -m/1000 = \ln\frac{1}{2} \quad \Rightarrow \quad m = -1000\ln\frac{1}{2} = 1000\ln 2 \approx 693.1$ h.