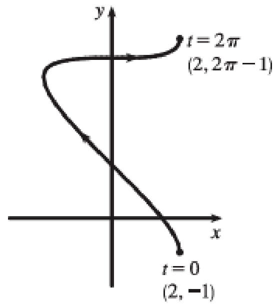
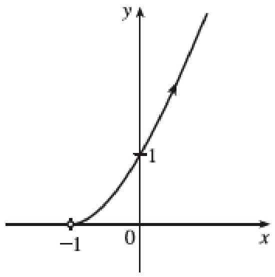
2. $x = 2\cos t, \quad y = t - \cos t, \quad 0 \le t \le 2\pi$

t	0	$\pi/2$	π	$3\pi/2$	2π
x	2	0	-2	0	2
y	-1	$\pi/2$	$\pi+1$	$3\pi/2$	$2\pi-1$
		1.57	4.14	4.71	5.28



14. (a) $x = e^t - 1, y = e^{2t}$. $\quad y = (e^t)^2 = (x+1)^2$ and since $x > -1$, we have the right side of the parabola $y = (x+1)^2$.

(b)



24. (a) From the first graph, we have $1 \le x \le 2$. From the second graph, we have $-1 \le y \le 1$. The only choice that satisfies either of those conditions is III.

(b) From the first graph, the values of x cycle through the values from -2 to 2 four times. From the second graph, the values of y cycle through the values from -2 to 2 six times. Choice I satisfies these conditions.

(c) From the first graph, the values of x cycle through the values from -2 to 2 three times. From the second graph, we have $0 \le y \le 2$. Choice IV satisfies these conditions.

(d) From the first graph, the values of x cycle through the values from -2 to 2 two times. From the second graph, the values of y do the same thing. Choice II satisfies these conditions.

28. (a) $x = t^4 - t + 1 = (t^4+1) - t > 0$ [think of the graphs of $y = t^4 + 1$ and $y = t$] and $y = t^2 \ge 0$, so these equations are matched with graph V.

(b) $y = \sqrt{t} \ge 0$. $\quad x = t^2 - 2t = t(t-2)$ is negative for $0 < t < 2$, so these equations are matched with graph I.

(c) $x = \sin 2t$ has period $2\pi/2 = \pi$. Note that

$y(t+2\pi) = \sin[t + 2\pi + \sin 2(t+2\pi)] = \sin(t + 2\pi + \sin 2t) = \sin(t + \sin 2t) = y(t)$, so y has period 2π.

These equations match graph II since x cycles through the values -1 to 1 twice as y cycles through those values once.

(d) $x = \cos 5t$ has period $2\pi/5$ and $y = \sin 2t$ has period π, so x will take on the values -1 to 1, and then 1 to -1, before y takes on the values -1 to 1. Note that when $t = 0$, $(x, y) = (1, 0)$. These equations are matched with graph VI.

(e) $x = t + \sin 4t, \; y = t^2 + \cos 3t$. As t becomes large, t and t^2 become the dominant terms in the expressions for x and y, so the graph will look like the graph of $y = x^2$, but with oscillations. These equations are matched with graph IV.

(f) $x = \dfrac{\sin 2t}{4+t^2}, \; y = \dfrac{\cos 2t}{4+t^2}$. As $t \to \infty$, x and y both approach 0. These equations are matched with graph III.

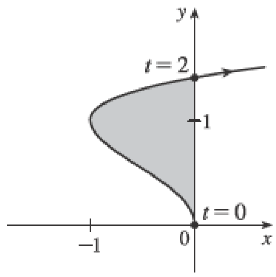
11.2

6. $x = \cos\theta + \sin 2\theta$, $y = \sin\theta + \cos 2\theta$; $\theta = 0$. $\dfrac{dy}{dx} = \dfrac{dy/d\theta}{dx/d\theta} = \dfrac{\cos\theta - 2\sin 2\theta}{-\sin\theta + 2\cos 2\theta}$. When $\theta = 0$, $(x, y) = (1, 1)$ and $dy/dx = \frac{1}{2}$, so an equation of the tangent to the curve is $y - 1 = \frac{1}{2}(x - 1)$, or $y = \frac{1}{2}x + \frac{1}{2}$.

14. $x = t + \ln t$, $y = t - \ln t \;\Rightarrow\; \dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{1 - 1/t}{1 + 1/t} = \dfrac{t-1}{t+1} = 1 - \dfrac{2}{t+1} \;\Rightarrow$

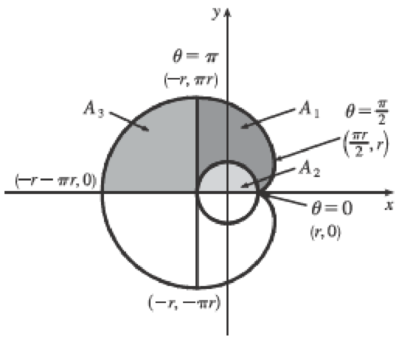
$\dfrac{d^2y}{dx^2} = \dfrac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{dx/dt} = \dfrac{\frac{d}{dt}\left(1 - \frac{2}{t+1}\right)}{1 + 1/t} = \dfrac{2/(t+1)^2}{(t+1)/t} = \dfrac{2t}{(t+1)^3}$, so the curve is CU for all t in its domain, that is, $t > 0$ [$t < -1$ not in domain].

32. The curve $x = t^2 - 2t = t(t-2)$, $y = \sqrt{t}$ intersects the y-axis when $x = 0$, that is, when $t = 0$ and $t = 2$. The corresponding values of y are 0 and $\sqrt{2}$. The shaded area is given by



$$\int_{y=0}^{y=\sqrt{2}} (x_R - x_L)\,dy = \int_{t=0}^{t=2} [0 - x(t)]\,y'(t)\,dt = -\int_0^2 (t^2 - 2t)\left(\frac{1}{2\sqrt{t}}\,dt\right)$$
$$= -\int_0^2 \left(\tfrac{1}{2}t^{3/2} - t^{1/2}\right) dt = -\left[\tfrac{1}{5}t^{5/2} - \tfrac{2}{3}t^{3/2}\right]_0^2$$
$$= -\left(\tfrac{1}{5}\cdot 2^{5/2} - \tfrac{2}{3}\cdot 2^{3/2}\right) = -2^{1/2}\left(\tfrac{4}{5} - \tfrac{4}{3}\right) = -\sqrt{2}\left(-\tfrac{8}{15}\right) = \tfrac{8}{15}\sqrt{2}$$

74. If the cow walks with the rope taut, it traces out the portion of the involute in Exercise 73 corresponding to the range $0 \le \theta \le \pi$, arriving at the point $(-r, \pi r)$ when $\theta = \pi$. With the rope now fully extended, the cow walks in a semicircle of radius πr, arriving at $(-r, -\pi r)$. Finally, the cow traces out another portion of the involute, namely the reflection about the x-axis of the initial involute path. (This corresponds to the range $-\pi \le \theta \le 0$.) Referring to the figure, we see that the total grazing area is $2(A_1 + A_3)$. A_3 is one-quarter of the area of a circle of radius πr, so $A_3 = \frac{1}{4}\pi(\pi r)^2 = \frac{1}{4}\pi^3 r^2$. We will compute $A_1 + A_2$ and then subtract $A_2 = \frac{1}{2}\pi r^2$ to obtain A_1.



To find $A_1 + A_2$, first note that the rightmost point of the involute is $\left(\frac{\pi}{2}r, r\right)$. [To see this, note that $dx/d\theta = 0$ when $\theta = 0$ or $\frac{\pi}{2}$. $\theta = 0$ corresponds to the cusp at $(r, 0)$ and $\theta = \frac{\pi}{2}$ corresponds to $\left(\frac{\pi}{2}r, r\right)$.] The leftmost point of the involute is $(-r, \pi r)$. Thus, $A_1 + A_2 = \int_{\theta=\pi}^{\pi/2} y\,dx - \int_{\theta=0}^{\pi/2} y\,dx = \int_{\theta=\pi}^{0} y\,dx$.

Now $y\,dx = r(\sin\theta - \theta\cos\theta)\,r\theta\cos\theta\,d\theta = r^2(\theta\sin\theta\,\cos\theta - \theta^2\cos^2\theta)d\theta$. Integrate:

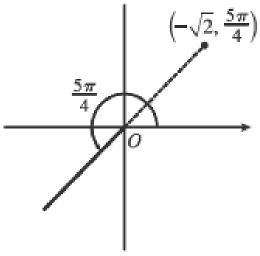
$(1/r^2)\int y\,dx = -\theta\cos^2\theta - \frac{1}{2}\left(\theta^2 - 1\right)\sin\theta\,\cos\theta - \frac{1}{6}\theta^3 + \frac{1}{2}\theta + C$. This enables us to compute

$$A_1 + A_2 = r^2\left[-\theta\cos^2\theta - \tfrac{1}{2}(\theta^2 - 1)\sin\theta\,\cos\theta - \tfrac{1}{6}\theta^3 + \tfrac{1}{2}\theta\right]_\pi^0 = r^2\left[0 - \left(-\pi - \frac{\pi^3}{6} + \frac{\pi}{2}\right)\right] = r^2\left(\frac{\pi}{2} + \frac{\pi^3}{6}\right)$$

Therefore, $A_1 = (A_1 + A_2) - A_2 = \frac{1}{6}\pi^3 r^2$, so the grazing area is $2(A_1 + A_3) = 2\left(\frac{1}{6}\pi^3 r^2 + \frac{1}{4}\pi^3 r^2\right) = \frac{5}{6}\pi^3 r^2$.

11.3

4. (a)

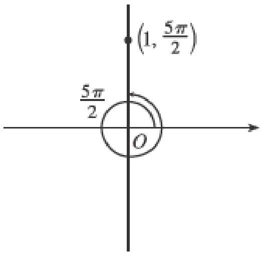


$x = -\sqrt{2}\cos\frac{5\pi}{4} = -\sqrt{2}\left(-\frac{\sqrt{2}}{2}\right) = 1$ and

$y = -\sqrt{2}\sin\frac{5\pi}{4} = -\sqrt{2}\left(-\frac{\sqrt{2}}{2}\right) = 1$

gives us $(1, 1)$.

(b)

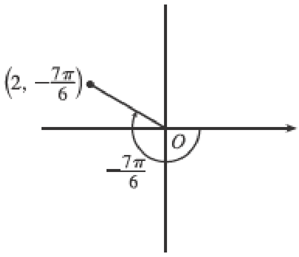


$x = 1\cos\frac{5\pi}{2} = 1(0) = 0$ and

$y = 1\sin\frac{5\pi}{2} = 1(1) = 1$

gives us $(0, 1)$.

(c)

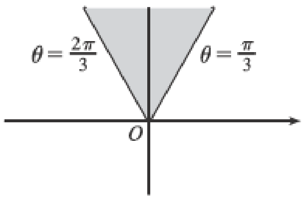


$x = 2\cos\left(-\frac{7\pi}{6}\right) = 2\left(-\frac{\sqrt{3}}{2}\right) = -\sqrt{3}$ and

$y = 2\sin\left(-\frac{7\pi}{6}\right) = 2\left(\frac{1}{2}\right) = 1$

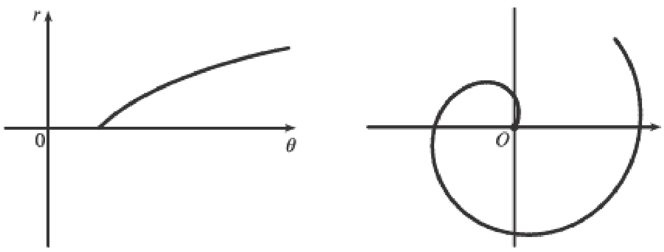
give us $\left(-\sqrt{3}, 1\right)$.

8. $r \geq 0, \quad \pi/3 \leq \theta \leq 2\pi/3$



16. $r\cos\theta = 1 \quad \Leftrightarrow \quad x = 1$, a vertical line.

36. $r = \ln\theta, \ \theta \geq 1$



56. (a) $r = \sqrt{\theta},\ 0 \le \theta \le 16\pi$. r increases as θ increases and there are eight full revolutions. The graph must be either II or V. When $\theta = 2\pi$, $r = \sqrt{2\pi} \approx 2.5$ and when $\theta = 16\pi$, $r = \sqrt{16\pi} \approx 7$, so the last revolution intersects the polar axis at approximately 3 times the distance that the first revolution intersects the polar axis, which is depicted in graph V.

(b) $r = \theta^2,\ 0 \le \theta \le 16\pi$. See part (a). This is graph II.

(c) $r = \cos(\theta/3)$. $0 \le \frac{\theta}{3} \le 2\pi \;\Rightarrow\; 0 \le \theta \le 6\pi$, so this curve will repeat itself every 6π radians.
$\cos\left(\frac{\theta}{3}\right) = 0 \;\Rightarrow\; \frac{\theta}{3} = \frac{\pi}{2} + \pi n \;\Rightarrow\; \theta = \frac{3\pi}{2} + 3\pi n$, so there will be two "pole" values, $\frac{3\pi}{2}$ and $\frac{9\pi}{2}$.
This is graph VI.

(d) $r = 1 + 2\cos\theta$ is a limaçon [see Exercise 55(a)] with $c = 2$. This is graph III.

(e) Since $-1 \le \sin 3\theta \le 1$, $1 \le 2 + \sin 3\theta \le 3$, so $r = 2 + \sin 3\theta$ is never 0; that is, the curve never intersects the pole. This is graph I.

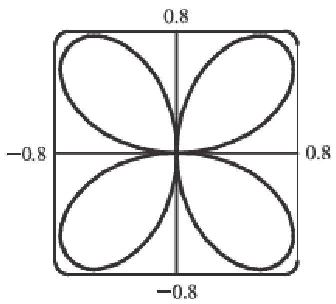
(f) $r = 1 + 2\sin 3\theta$. Solving $r = 0$ will give us many "pole" values, so this is graph IV.

58. $r = 2 - \sin\theta \;\Rightarrow\; x = r\cos\theta = (2 - \sin\theta)\cos\theta$, $y = r\sin\theta = (2 - \sin\theta)\sin\theta \;\Rightarrow$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{(2-\sin\theta)\cos\theta + \sin\theta(-\cos\theta)}{(2-\sin\theta)(-\sin\theta) + \cos\theta(-\cos\theta)} = \frac{2\cos\theta - 2\sin\theta\,\cos\theta}{-2\sin\theta + \sin^2\theta - \cos^2\theta} = \frac{2\cos\theta - \sin 2\theta}{-2\sin\theta - \cos 2\theta}$$

When $\theta = \frac{\pi}{3}$, $\dfrac{dy}{dx} = \dfrac{2(1/2) - (\sqrt{3}/2)}{-2(\sqrt{3}/2) - (-1/2)} = \dfrac{1 - \sqrt{3}/2}{-\sqrt{3} + 1/2} \cdot \dfrac{2}{2} = \dfrac{2 - \sqrt{3}}{1 - 2\sqrt{3}}$.

78.



From the graph, the highest points seem to have $y \approx 0.77$. To find the exact value, we solve $dy/d\theta = 0$. $y = r\sin\theta = \sin\theta\,\sin 2\theta \;\Rightarrow$

$$\begin{aligned} dy/d\theta &= 2\sin\theta\,\cos 2\theta + \cos\theta\,\sin 2\theta \\ &= 2\sin\theta\,(2\cos^2\theta - 1) + \cos\theta\,(2\sin\theta\,\cos\theta) \\ &= 2\sin\theta\,(3\cos^2\theta - 1) \end{aligned}$$

In the first quadrant, this is 0 when $\cos\theta = \frac{1}{\sqrt{3}} \;\Leftrightarrow\; \sin\theta = \sqrt{\frac{2}{3}} \;\Leftrightarrow$
$y = 2\sin^2\theta\cos\theta = 2 \cdot \frac{2}{3} \cdot \frac{1}{\sqrt{3}} = \frac{4}{9}\sqrt{3} \approx 0.77$.

11.4

6. $r = 1 + \cos\theta,\ 0 \le \theta \le \pi$.

$$A = \int_0^{\pi} \tfrac{1}{2}(1+\cos\theta)^2\,d\theta = \tfrac{1}{2}\int_0^{\pi}(1 + 2\cos\theta + \cos^2\theta)\,d\theta = \tfrac{1}{2}\int_0^{\pi}\left[1 + 2\cos\theta + \tfrac{1}{2}(1+\cos 2\theta)\right]d\theta$$

$$= \tfrac{1}{2}\int_0^{\pi}\left(\tfrac{3}{2} + 2\cos\theta + \tfrac{1}{2}\cos 2\theta\right)d\theta = \tfrac{1}{2}\left[\tfrac{3}{2}\theta + 2\sin\theta + \tfrac{1}{4}\sin 2\theta\right]_0^{\pi} = \tfrac{1}{2}\left(\tfrac{3}{2}\pi + 0 + 0\right) - \tfrac{1}{2}(0) = \tfrac{3\pi}{4}$$

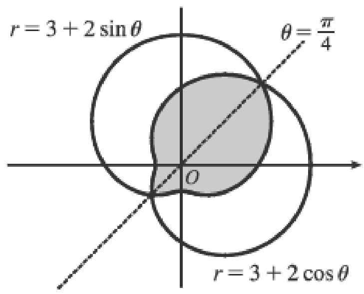
32. $3 + 2\cos\theta = 3 + 2\sin\theta \;\Rightarrow\; \cos\theta = \sin\theta \;\Rightarrow\; \theta = \frac{\pi}{4}$ or $\frac{5\pi}{4}$.

$A = 2\int_{\pi/4}^{5\pi/4} \frac{1}{2}(3 + 2\cos\theta)^2\,d\theta = \int_{\pi/4}^{5\pi/4}(9 + 12\cos\theta + 4\cos^2\theta)\,d\theta$

$= \int_{\pi/4}^{5\pi/4}\left[9 + 12\cos\theta + 4\cdot\frac{1}{2}(1+\cos 2\theta)\right]d\theta$

$= \int_{\pi/4}^{5\pi/4}(11 + 12\cos\theta + 2\cos 2\theta)\,d\theta = \left[11\theta + 12\sin\theta + \sin 2\theta\right]_{\pi/4}^{5\pi/4}$

$= \left(\frac{55\pi}{4} - 6\sqrt{2} + 1\right) - \left(\frac{11\pi}{4} + 6\sqrt{2} + 1\right) = 11\pi - 12\sqrt{2}$

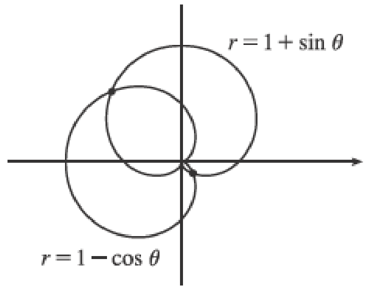


38. The pole is a point of intersection.

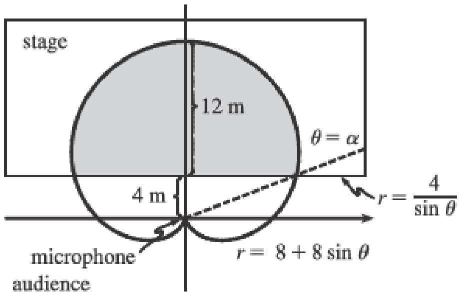
$1 - \cos\theta = 1 + \sin\theta \quad \Rightarrow \quad -\cos\theta = \sin\theta \quad \Rightarrow \quad -1 = \tan\theta \quad \Rightarrow$

$\theta = \frac{3\pi}{4}$ or $\frac{7\pi}{4}$.

The other two points of intersection are $\left(1 + \frac{\sqrt{2}}{2}, \frac{3\pi}{4}\right)$ and $\left(1 - \frac{\sqrt{2}}{2}, \frac{7\pi}{4}\right)$.



44.



We need to find the shaded area A in the figure. The horizontal line representing the front of the stage has equation $y = 4 \quad \Leftrightarrow$ $r\sin\theta = 4 \quad \Rightarrow \quad r = 4/\sin\theta$. This line intersects the curve $r = 8 + 8\sin\theta$ when $8 + 8\sin\theta = \dfrac{4}{\sin\theta} \quad \Rightarrow$ $8\sin\theta + 8\sin^2\theta = 4 \quad \Rightarrow \quad 2\sin^2\theta + 2\sin\theta - 1 = 0 \quad \Rightarrow$

$\sin\theta = \dfrac{-2 \pm \sqrt{4+8}}{4} = \dfrac{-2 \pm 2\sqrt{3}}{4} = \dfrac{-1+\sqrt{3}}{2}$ [the other value is less than -1] $\Rightarrow \quad \theta = \sin^{-1}\left(\dfrac{\sqrt{3}-1}{2}\right)$.

This angle is about $21.5°$ and is denoted by α in the figure.

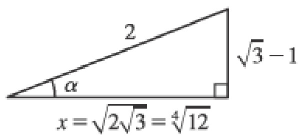
$$A = 2\int_\alpha^{\pi/2} \tfrac{1}{2}(8 + 8\sin\theta)^2\,d\theta - 2\int_\alpha^{\pi/2} \tfrac{1}{2}(4\csc\theta)^2\,d\theta = 64\int_\alpha^{\pi/2}(1 + 2\sin\theta + \sin^2\theta)\,d\theta - 16\int_\alpha^{\pi/2}\csc^2\theta\,d\theta$$

$$= 64\int_\alpha^{\pi/2}\left(1 + 2\sin\theta + \tfrac{1}{2} - \tfrac{1}{2}\cos 2\theta\right)\,d\theta + 16\int_\alpha^{\pi/2}(-\csc^2\theta)\,d\theta = 64\left[\tfrac{3}{2}\theta - 2\cos\theta - \tfrac{1}{4}\sin 2\theta\right]_\alpha^{\pi/2} + 16\left[\cot\theta\right]_\alpha^{\pi/2}$$

$$= 16\left[6\theta - 8\cos\theta - \sin 2\theta + \cot\theta\right]_\alpha^{\pi/\alpha} = 16[(3\pi - 0 - 0 + 0) - (6\alpha - 8\cos\alpha - \sin 2\alpha + \cot\alpha)]$$

$$= 48\pi - 96\alpha + 128\cos\alpha + 16\sin 2\alpha - 16\cot\alpha$$

From the figure, $x^2 + \left(\sqrt{3} - 1\right)^2 = 2^2 \quad \Rightarrow \quad x^2 = 4 - \left(3 - 2\sqrt{3} + 1\right) \quad \Rightarrow$ $x^2 = 2\sqrt{3} = \sqrt{12}$, so $x = \sqrt{2\sqrt{3}} = \sqrt[4]{12}$. Using the trigonometric relationships for a right triangle and the identity $\sin 2\alpha = 2\sin\alpha\cos\alpha$, we continue:



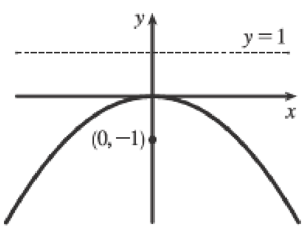
$$A = 48\pi - 96\alpha + 128\cdot\frac{\sqrt[4]{12}}{2} + 16\cdot 2\cdot\frac{\sqrt{3}-1}{2}\cdot\frac{\sqrt[4]{12}}{2} - 16\cdot\frac{\sqrt[4]{12}}{\sqrt{3}-1}\cdot\frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= 48\pi - 96\alpha + 64\sqrt[4]{12} + 8\sqrt[4]{12}\left(\sqrt{3}-1\right) - 8\sqrt[4]{12}\left(\sqrt{3}+1\right) = 48\pi + 48\sqrt[4]{12} - 96\sin^{-1}\left(\frac{\sqrt{3}-1}{2}\right)$$

$$\approx 204.16\ \text{m}^2$$

11.5

2. $4y + x^2 = 0 \quad \Rightarrow \quad x^2 = -4y$. $4p = -4$, so $p = -1$.
The vertex is $(0,0)$, the focus is $(0,-1)$, and the directrix is $y = 1$.



18. The ellipse is centered at $(2,1)$, with $a = 3$ and $b = 2$. An equation is $\dfrac{(x-2)^2}{9} + \dfrac{(y-1)^2}{4} = 1$. $c = \sqrt{a^2 - b^2} = \sqrt{5}$, so the foci are $\left(2 \pm \sqrt{5}, 1\right)$.

34. The distance from the focus $(3, 6)$ to the vertex $(3, 2)$ is $6 - 2 = 4$. Since the focus is above the vertex, $p = 4$. An equation is $(x-3)^2 = 4p(y-2) \Rightarrow (x-3)^2 = 16(y-2)$.

52. $|PF_1| - |PF_2| = \pm 2a \;\Leftrightarrow\; \sqrt{(x+c)^2 + y^2} - \sqrt{(x-c)^2 + y^2} = \pm 2a \;\Leftrightarrow$

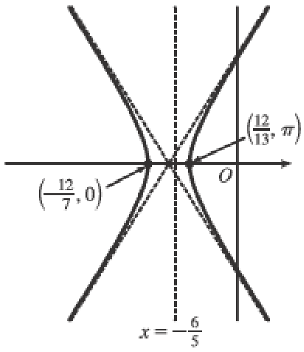
$\sqrt{(x+c)^2 + y^2} = \sqrt{(x-c)^2 + y^2} \pm 2a \;\Leftrightarrow\; (x+c)^2 + y^2 = (x-c)^2 + y^2 + 4a^2 \pm 4a\sqrt{(x-c)^2 + y^2} \;\Leftrightarrow$

$4cx - 4a^2 = \pm 4a\sqrt{(x-c)^2 + y^2} \;\Leftrightarrow\; c^2x^2 - 2a^2cx + a^4 = a^2(x^2 - 2cx + c^2 + y^2) \;\Leftrightarrow$

$(c^2 - a^2)x^2 - a^2y^2 = a^2(c^2 - a^2) \;\Leftrightarrow\; b^2x^2 - a^2y^2 = a^2b^2$ [where $b^2 = c^2 - a^2$] $\Leftrightarrow\; \dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$

11.6

10. $r = \dfrac{12}{3 - 10\cos\theta} \cdot \dfrac{1/3}{1/3} = \dfrac{4}{1 - \frac{10}{3}\cos\theta}$, where $e = \frac{10}{3}$ and $ed = 4 \Rightarrow d = 4\left(\frac{3}{10}\right) = \frac{6}{5}$.



(a) Eccentricity $= e = \frac{10}{3}$

(b) Since $e = \frac{10}{3} > 1$, the conic is a hyperbola.

(c) Since "$-e\cos\theta$" appears in the denominator, the directrix is to the left of the focus at the origin. $d = |Fl| = \frac{6}{5}$, so an equation of the directrix is $x = -\frac{6}{5}$.

(d) The vertices are $\left(-\frac{12}{7}, 0\right)$ and $\left(\frac{12}{13}, \pi\right)$, so the center is midway between them, that is, $\left(\frac{120}{91}, \pi\right)$.

26. We are given $e = 0.048$ and $2a = 1.56 \times 10^9 \Rightarrow a = 7.8 \times 10^8$. By (7), we have

$$r = \frac{a(1-e^2)}{1 + e\cos\theta} = \frac{7.8 \times 10^8[1 - (0.048)^2]}{1 + 0.048\cos\theta} \approx \frac{7.78 \times 10^8}{1 + 0.048\cos\theta}$$