INCREASING AND DECREASING FUNCTIONS

The graph shown in Figure 22 rises from A to B, falls from B to C, and rises again from C to D. The function f is said to be increasing on the interval $[a, b]$, decreasing on $[b, c]$, and increasing again on $[c, d]$. Notice that if x_1 and x_2 are any two numbers between a and b with $x_1 < x_2$, then $f(x_1) < f(x_2)$. We use this as the defining property of an increasing function.

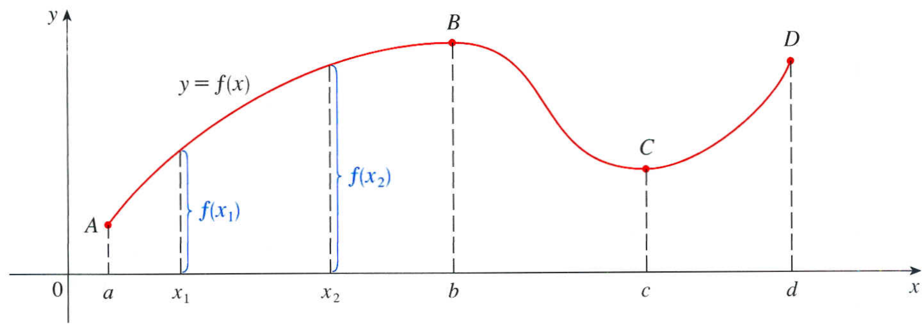


FIGURE 22

A function f is called **increasing** on an interval I if

$$f(x_1) < f(x_2) \qquad \text{whenever } x_1 < x_2 \text{ in } I$$

It is called **decreasing** on I if

$$f(x_1) > f(x_2) \qquad \text{whenever } x_1 < x_2 \text{ in } I$$

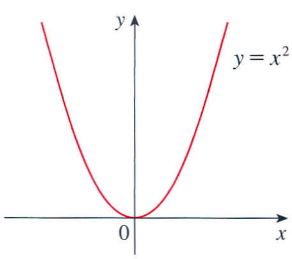


FIGURE 23

In the definition of an increasing function it is important to realize that the inequality $f(x_1) < f(x_2)$ must be satisfied for *every* pair of numbers x_1 and x_2 in I with $x_1 < x_2$.

You can see from Figure 23 that the function $f(x) = x^2$ is decreasing on the interval $(-\infty, 0]$ and increasing on the interval $[0, \infty)$.

1.1 EXERCISES

1. The graph of a function f is given.

(a) State the value of $f(-1)$.

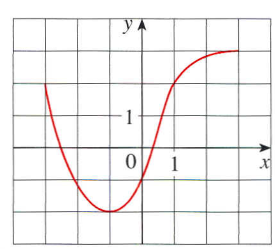
(b) Estimate the value of $f(2)$.

(c) For what values of x is $f(x) = 2$?

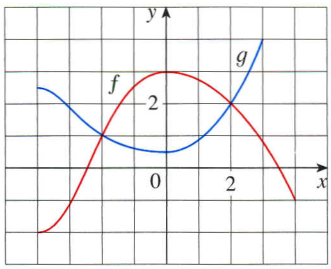
(d) Estimate the values of x such that $f(x) = 0$.

(e) State the domain and range of f.

(f) On what interval is f increasing?



2. The graphs of f and g are given.
   (a) State the values of $f(-4)$ and $g(3)$.
   (b) For what values of x is $f(x) = g(x)$?
   (c) Estimate the solution of the equation $f(x) = -1$.
   (d) On what interval is f decreasing?
   (e) State the domain and range of f.
   (f) State the domain and range of g.

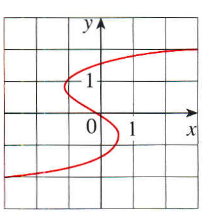


3. Figure 1 was recorded by an instrument operated by the California Department of Mines and Geology at the University Hospital of the University of Southern California in Los Angeles. Use it to estimate the range of the vertical ground acceleration function at USC during the Northridge earthquake.

4. In this section we discussed examples of ordinary, everyday functions: Population is a function of time, postage cost is a function of weight, water temperature is a function of time. Give three other examples of functions from everyday life that are described verbally. What can you say about the domain and range of each of your functions? If possible, sketch a rough graph of each function.

5–8 Determine whether the curve is the graph of a function of x. If it is, state the domain and range of the function.

5.



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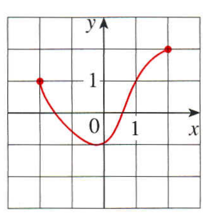

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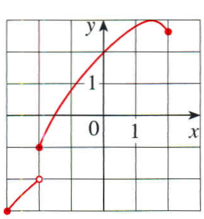

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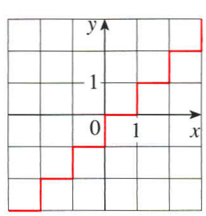
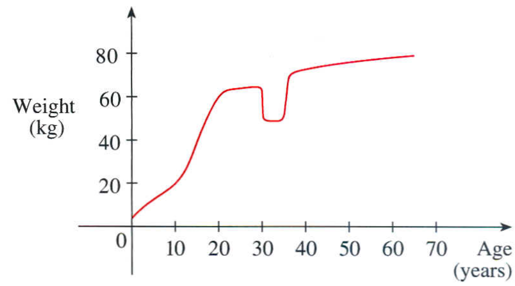
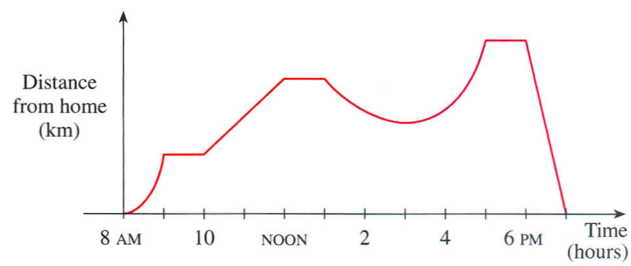


9. The graph shown gives the weight of a certain person as a function of age. Describe in words how this person's weight varies over time. What do you think happened when this person was 30 years old?



10. The graph shown gives a salesman's distance from his home as a function of time on a certain day. Describe in words what the graph indicates about his travels on this day.



11. You put some ice cubes in a glass, fill the glass with cold water, and then let the glass sit on a table. Describe how the temperature of the water changes as time passes. Then sketch a rough graph of the temperature of the water as a function of the elapsed time.

12. Sketch a rough graph of the number of hours of daylight as a function of the time of year.

13. Sketch a rough graph of the outdoor temperature as a function of time during a typical spring day.

14. Sketch a rough graph of the market value of a new car as a function of time for a period of 20 years. Assume the car is well maintained.

15. Sketch the graph of the amount of a particular brand of coffee sold by a store as a function of the price of the coffee.

16. You place a frozen pie in an oven and bake it for an hour. Then you take it out and let it cool before eating it. Describe how the temperature of the pie changes as time passes. Then sketch a rough graph of the temperature of the pie as a function of time.

17. A homeowner mows the lawn every Wednesday afternoon. Sketch a rough graph of the height of the grass as a function of time over the course of a four-week period.

18. An airplane flies from an airport and lands an hour later at another airport, 400 km away. If t represents the time in minutes since the plane has left the terminal building, let $x(t)$ be

the horizontal distance traveled and $y(t)$ be the altitude of the plane.

(a) Sketch a possible graph of $x(t)$.

(b) Sketch a possible graph of $y(t)$.

(c) Sketch a possible graph of the ground speed.

(d) Sketch a possible graph of the vertical velocity.

19. The number N (in millions) of cellular phone subscribers worldwide is shown in the table. (Midyear estimates are given.)

t	1990	1992	1994	1996	1998	2000
N	11	26	60	160	340	650

(a) Use the data to sketch a rough graph of N as a function of t.

(b) Use your graph to estimate the number of cell-phone subscribers at midyear in 1995 and 1999.

20. Temperature readings T (in °C) were recorded every three hours from midnight to 3:00 PM in Montréal on July 13, 2004. The time t was measured in hours from midnight.

t	0	3	6	9	12	15
T	21.5	19.8	20.0	22.2	24.8	25.8

(a) Use the readings to sketch a rough graph of T as a function of t.

(b) Use your graph to estimate the temperature at 11:00 AM.

21. If $f(x) = 3x^2 - x + 2$, find $f(2)$, $f(-2)$, $f(a)$, $f(-a)$, $f(a + 1)$, $2f(a)$, $f(2a)$, $f(a^2)$, $[f(a)]^2$, and $f(a + h)$.

22. A spherical balloon with radius r centimeters has volume $V(r) = \frac{4}{3}\pi r^3$. Find a function that represents the amount of air required to inflate the balloon from a radius of r centimeters to a radius of $r + 1$ centimeters.

23–26 Evaluate the difference quotient for the given function. Simplify your answer.

23. $f(x) = 4 + 3x - x^2$, $\quad \dfrac{f(3 + h) - f(3)}{h}$

24. $f(x) = x^3$, $\quad \dfrac{f(a + h) - f(a)}{h}$

25. $f(x) = \dfrac{1}{x}$, $\quad \dfrac{f(x) - f(a)}{x - a}$

26. $f(x) = \dfrac{x + 3}{x + 1}$, $\quad \dfrac{f(x) - f(1)}{x - 1}$

27–31 Find the domain of the function.

27. $f(x) = \dfrac{x}{3x - 1}$

28. $f(x) = \dfrac{5x + 4}{x^2 + 3x + 2}$

29. $f(t) = \sqrt{t} + \sqrt[3]{t}$

30. $g(u) = \sqrt{u} + \sqrt{4 - u}$

31. $h(x) = \dfrac{1}{\sqrt[4]{x^2 - 5x}}$

32. Find the domain and range and sketch the graph of the function $h(x) = \sqrt{4 - x^2}$.

33–44 Find the domain and sketch the graph of the function.

33. $f(x) = 5$

34. $F(x) = \frac{1}{2}(x + 3)$

35. $f(t) = t^2 - 6t$

36. $H(t) = \dfrac{4 - t^2}{2 - t}$

37. $g(x) = \sqrt{x - 5}$

38. $F(x) = |2x + 1|$

39. $G(x) = \dfrac{3x + |x|}{x}$

40. $g(x) = \dfrac{|x|}{x^2}$

41. $f(x) = \begin{cases} x + 2 & \text{if } x < 0 \\ 1 - x & \text{if } x \geqslant 0 \end{cases}$

42. $f(x) = \begin{cases} 3 - \frac{1}{2}x & \text{if } x \leqslant 2 \\ 2x - 5 & \text{if } x > 2 \end{cases}$

43. $f(x) = \begin{cases} x + 2 & \text{if } x \leqslant -1 \\ x^2 & \text{if } x > -1 \end{cases}$

44. $f(x) = \begin{cases} x + 9 & \text{if } x < -3 \\ -2x & \text{if } |x| \leqslant 3 \\ -6 & \text{if } x > 3 \end{cases}$

45–50 Find an expression for the function whose graph is the given curve.

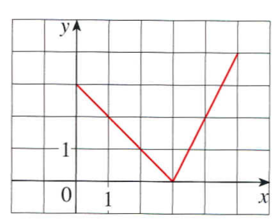
45. The line segment joining the points $(1, -3)$ and $(5, 7)$

46. The line segment joining the points $(-5, 10)$ and $(7, -10)$

47. The bottom half of the parabola $x + (y - 1)^2 = 0$

48. The top half of the circle $x^2 + (y - 2)^2 = 4$

49.



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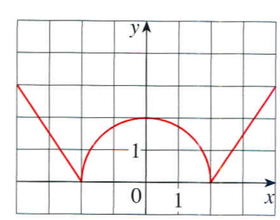

51–55 Find a formula for the described function and state its domain.

51. A rectangle has perimeter 20 m. Express the area of the rectangle as a function of the length of one of its sides.

52. A rectangle has area 16 m^2. Express the perimeter of the rectangle as a function of the length of one of its sides.

53. Express the area of an equilateral triangle as a function of the length of a side.

54. Express the surface area of a cube as a function of its volume.

55. An open rectangular box with volume 2 m^3 has a square base. Express the surface area of the box as a function of the length of a side of the base.

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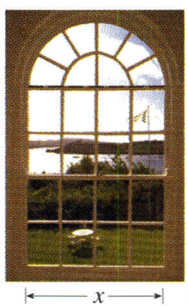
56. A Norman window has the shape of a rectangle surmounted by a semicircle. If the perimeter of the window is 10 m, express the area A of the window as a function of the width x of the window.



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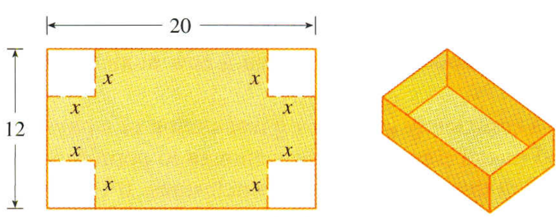
57. A box with an open top is to be constructed from a rectangular piece of cardboard with dimensions 12 cm by 20 cm by cutting out equal squares of side x at each corner and then folding up the sides as in the figure. Express the volume V of the box as a function of x.



58. A taxi company charges two dollars for the first kilometer (or part of a kilometer) and 20 cents for each succeeding tenth of a kilometer (or part). Express the cost C (in dollars) of a ride as a function of the distance x traveled (in kilometers) for $0 < x < 2$, and sketch the graph of this function.

59. In a certain country, income tax is assessed as follows. There is no tax on income up to \$10,000. Any income over \$10,000 is taxed at a rate of 10%, up to an income of \$20,000. Any income over \$20,000 is taxed at 15%.

(a) Sketch the graph of the tax rate R as a function of the income I.

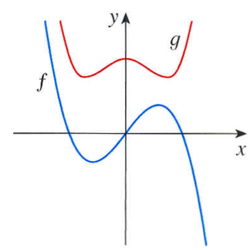
(b) How much tax is assessed on an income of \$14,000? On \$26,000?

(c) Sketch the graph of the total assessed tax T as a function of the income I.

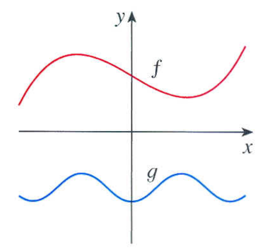
60. The functions in Example 10 and Exercises 58 and 59(a) are called *step functions* because their graphs look like stairs. Give two other examples of step functions that arise in everyday life.

61–62 Graphs of f and g are shown. Decide whether each function is even, odd, or neither. Explain your reasoning.

61.



62.



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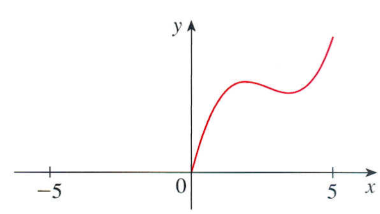
63. (a) If the point (5, 3) is on the graph of an even function, what other point must also be on the graph?

(b) If the point (5, 3) is on the graph of an odd function, what other point must also be on the graph?

64. A function f has domain $[-5, 5]$ and a portion of its graph is shown.

(a) Complete the graph of f if it is known that f is even.

(b) Complete the graph of f if it is known that f is odd.



65–70 Determine whether f is even, odd, or neither. If you have a graphing calculator, use it to check your answer visually.

65. $f(x) = \dfrac{x}{x^2 + 1}$

66. $f(x) = \dfrac{x^2}{x^4 + 1}$

67. $f(x) = \dfrac{x}{x + 1}$

68. $f(x) = x|x|$

69. $f(x) = 1 + 3x^2 - x^4$

70. $f(x) = 1 + 3x^3 - x^5$

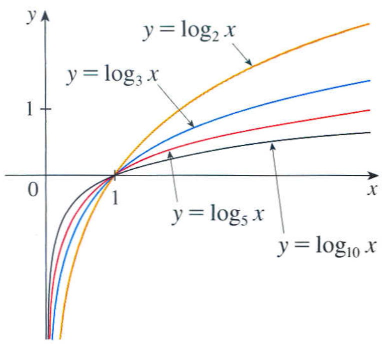


FIGURE 21

LOGARITHMIC FUNCTIONS

The **logarithmic functions** $f(x) = \log_a x$, where the base a is a positive constant, are the inverse functions of the exponential functions. They will be studied in Section 1.6. Figure 21 shows the graphs of four logarithmic functions with various bases. In each case the domain is $(0, \infty)$, the range is $(-\infty, \infty)$, and the function increases slowly when $x > 1$.

TRANSCENDENTAL FUNCTIONS

These are functions that are not algebraic. The set of transcendental functions includes the trigonometric, inverse trigonometric, exponential, and logarithmic functions, but it also includes a vast number of other functions that have never been named. In Chapter 11 we will study transcendental functions that are defined as sums of infinite series.

EXAMPLE 5 Classify the following functions as one of the types of functions that we have discussed.

(a) $f(x) = 5^x$ (b) $g(x) = x^5$

(c) $h(x) = \dfrac{1 + x}{1 - \sqrt{x}}$ (d) $u(t) = 1 - t + 5t^4$

SOLUTION

(a) $f(x) = 5^x$ is an exponential function. (The x is the exponent.)

(b) $g(x) = x^5$ is a power function. (The x is the base.) We could also consider it to be a polynomial of degree 5.

(c) $h(x) = \dfrac{1 + x}{1 - \sqrt{x}}$ is an algebraic function.

(d) $u(t) = 1 - t + 5t^4$ is a polynomial of degree 4. □

1.2 EXERCISES

1–2 Classify each function as a power function, root function, polynomial (state its degree), rational function, algebraic function, trigonometric function, exponential function, or logarithmic function.

1. (a) $f(x) = \sqrt[5]{x}$ (b) $g(x) = \sqrt{1 - x^2}$

(c) $h(x) = x^9 + x^4$ (d) $r(x) = \dfrac{x^2 + 1}{x^3 + x}$

(e) $s(x) = \tan 2x$ (f) $t(x) = \log_{10} x$

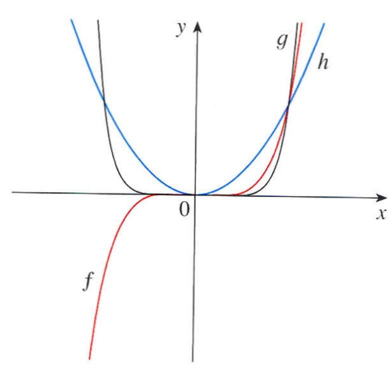
2. (a) $y = \dfrac{x - 6}{x + 6}$ (b) $y = x + \dfrac{x^2}{\sqrt{x - 1}}$

(c) $y = 10^x$ (d) $y = x^{10}$

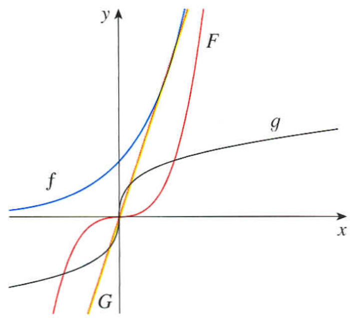
(e) $y = 2t^6 + t^4 - \pi$ (f) $y = \cos\theta + \sin\theta$

3–4 Match each equation with its graph. Explain your choices. (Don't use a computer or graphing calculator.)

3. (a) $y = x^2$ (b) $y = x^5$ (c) $y = x^8$



4. (a) $y = 3x$ (b) $y = 3^x$
(c) $y = x^3$ (d) $y = \sqrt[3]{x}$

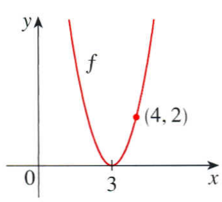


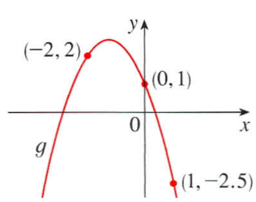
5. (a) Find an equation for the family of linear functions with slope 2 and sketch several members of the family.
(b) Find an equation for the family of linear functions such that $f(2) = 1$ and sketch several members of the family.
(c) Which function belongs to both families?

6. What do all members of the family of linear functions $f(x) = 1 + m(x + 3)$ have in common? Sketch several members of the family.

7. What do all members of the family of linear functions $f(x) = c - x$ have in common? Sketch several members of the family.

8. Find expressions for the quadratic functions whose graphs are shown.





9. Find an expression for a cubic function f if $f(1) = 6$ and $f(-1) = f(0) = f(2) = 0$.

10. Recent studies indicate that the average surface temperature of the earth has been rising steadily. Some scientists have modeled the temperature by the linear function $T = 0.02t + 8.50$, where T is temperature in °C and t represents years since 1900.
(a) What do the slope and T-intercept represent?
(b) Use the equation to predict the average global surface temperature in 2100.

11. If the recommended adult dosage for a drug is D (in mg), then to determine the appropriate dosage c for a child of age a, pharmacists use the equation $c = 0.0417D(a + 1)$. Suppose the dosage for an adult is 200 mg.
(a) Find the slope of the graph of c. What does it represent?
(b) What is the dosage for a newborn?

12. The manager of a weekend flea market knows from past experience that if he charges x dollars for a rental space at the market, then the number y of spaces he can rent is given by the equation $y = 200 - 4x$.
(a) Sketch a graph of this linear function. (Remember that the rental charge per space and the number of spaces rented can't be negative quantities.)
(b) What do the slope, the y-intercept, and the x-intercept of the graph represent?

13. The relationship between the Fahrenheit (F) and Celsius (C) temperature scales is given by the linear function $F = \frac{9}{5}C + 32$.
(a) Sketch a graph of this function.
(b) What is the slope of the graph and what does it represent? What is the F-intercept and what does it represent?

14. Kelly leaves Winnipeg at 2:00 PM and drives at a constant speed west along the Trans-Canada highway. He passes Brandon, 210 km from Winnipeg, at 4:00 PM.
(a) Express the distance traveled in terms of the time elapsed.
(b) Draw the graph of the equation in part (a).
(c) What is the slope of this line? What does it represent?

15. Biologists have noticed that the chirping rate of crickets of a certain species is related to temperature, and the relationship appears to be very nearly linear. A cricket produces 112 chirps per minute at 20°C and 180 chirps per minute at 29°C.
(a) Find a linear equation that models the temperature T as a function of the number of chirps per minute N.
(b) What is the slope of the graph? What does it represent?
(c) If the crickets are chirping at 150 chirps per minute, estimate the temperature.

16. The manager of a furniture factory finds that it costs \$2200 to manufacture 100 chairs in one day and \$4800 to produce 300 chairs in one day.
(a) Express the cost as a function of the number of chairs produced, assuming that it is linear. Then sketch the graph.
(b) What is the slope of the graph and what does it represent?
(c) What is the y-intercept of the graph and what does it represent?

17. At the surface of the ocean, the water pressure is the same as the air pressure above the water, 1.05 kg/cm^2. Below the surface, the water pressure increases by 0.10 kg/cm^2 for every meter of descent.
(a) Express the water pressure as a function of the depth below the ocean surface.
(b) At what depth is the pressure 7 kg/cm^2?

18. The monthly cost of driving a car depends on the number of kilometers driven. Lynn found that in May it cost her \$380 to drive 768 km and in June it cost her \$460 to drive 1280 km.

(a) Express the monthly cost C as a function of the distance driven d, assuming that a linear relationship gives a suitable model.

(b) Use part (a) to predict the cost of driving 2400 km per month.

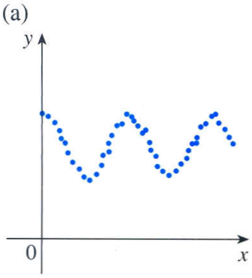
(c) Draw the graph of the linear function. What does the slope represent?

(d) What does the y-intercept represent?

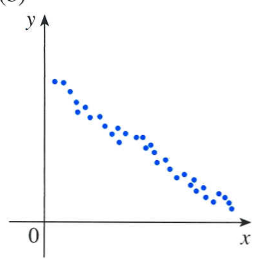
(e) Why does a linear function give a suitable model in this situation?

19–20 For each scatter plot, decide what type of function you might choose as a model for the data. Explain your choices.

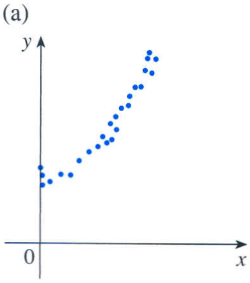
19. (a)



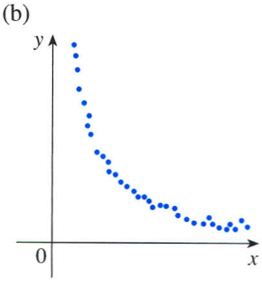
(b)



20. (a)



(b)



21. The table shows (lifetime) peptic ulcer rates (per 100 population) for various family incomes as reported by the National Health Interview Survey.

Income	Ulcer rate (per 100 population)
\$4,000	14.1
\$6,000	13.0
\$8,000	13.4
\$12,000	12.5
\$16,000	12.0
\$20,000	12.4
\$30,000	10.5
\$45,000	9.4
\$60,000	8.2

(a) Make a scatter plot of these data and decide whether a linear model is appropriate.

(b) Find and graph a linear model using the first and last data points.

(c) Find and graph the least squares regression line.

(d) Use the linear model in part (c) to estimate the ulcer rate for an income of \$25,000.

(e) According to the model, how likely is someone with an income of \$80,000 to suffer from peptic ulcers?

(f) Do you think it would be reasonable to apply the model to someone with an income of \$200,000?

22. Biologists have observed that the chirping rate of crickets of a certain species appears to be related to temperature. The table shows the chirping rates for various temperatures.

Temperature (°C)	Chirping rate (chirps/min)	Temperature (°C)	Chirping rate (chirps/min)
20	113	30	188
22	128	32	203
24	143	34	218
26	158	36	233
28	173		

(a) Make a scatter plot of the data.

(b) Find and graph the regression line.

(c) Use the linear model in part (b) to estimate the chirping rate at 40°C.

23. The table gives the winning heights for the Olympic pole vault competitions in the 20th century.

Year	Height (m)	Year	Height (m)
1900	3.30	1956	4.56
1904	3.50	1960	4.70
1908	3.71	1964	5.10
1912	3.95	1968	5.40
1920	4.09	1972	5.50
1924	3.95	1976	5.50
1928	4.20	1980	5.78
1932	4.31	1984	5.75
1936	4.35	1988	5.90
1948	4.30	1992	5.80
1952	4.55	1996	5.92

(a) Make a scatter plot and decide whether a linear model is appropriate.

(b) Find and graph the regression line.

(c) Use the linear model to predict the height of the winning pole vault at the 2000 Olympics and compare with the actual winning height of 5.90 m.

(d) Is it reasonable to use the model to predict the winning height at the 2100 Olympics?

24. The table shows the percentage of the population of Argentina that has lived in rural areas from 1955 to 2000. Find a model for the data and use it to estimate the rural percentage in 1988 and 2002.

Year	Percentage rural	Year	Percentage rural
1955	30.4	1980	17.1
1960	26.4	1985	15.0
1965	23.6	1990	13.0
1970	21.1	1995	11.7
1975	19.0	2000	10.5

25. Use the data in the table to model the population of the world in the 20th century by a cubic function. Then use your model to estimate the population in the year 1925.

Year	Population (millions)	Year	Population (millions)
1900	1650	1960	3040
1910	1750	1970	3710
1920	1860	1980	4450
1930	2070	1990	5280
1940	2300	2000	6080
1950	2560		

26. The table shows the mean (average) distances d of the planets from the sun (taking the unit of measurement to be the distance from the earth to the sun) and their periods T (time of revolution in years).

Planet	d	T
Mercury	0.387	0.241
Venus	0.723	0.615
Earth	1.000	1.000
Mars	1.523	1.881
Jupiter	5.203	11.861
Saturn	9.541	29.457
Uranus	19.190	84.008
Neptune	30.086	164.784

(a) Fit a power model to the data.

(b) Kepler's Third Law of Planetary Motion states that

> "The square of the period of revolution of a planet is proportional to the cube of its mean distance from the sun."

Does your model corroborate Kepler's Third Law?

1.3 NEW FUNCTIONS FROM OLD FUNCTIONS

In this section we start with the basic functions we discussed in Section 1.2 and obtain new functions by shifting, stretching, and reflecting their graphs. We also show how to combine pairs of functions by the standard arithmetic operations and by composition.

TRANSFORMATIONS OF FUNCTIONS

By applying certain transformations to the graph of a given function we can obtain the graphs of certain related functions. This will give us the ability to sketch the graphs of many functions quickly by hand. It will also enable us to write equations for given graphs. Let's first consider **translations**. If c is a positive number, then the graph of $y = f(x) + c$ is just the graph of $y = f(x)$ shifted upward a distance of c units (because each y-coordinate is increased by the same number c). Likewise, if $g(x) = f(x - c)$, where $c > 0$, then the value of g at x is the same as the value of f at $x - c$ (c units to the left of x). Therefore, the graph of $y = f(x - c)$ is just the graph of $y = f(x)$ shifted c units to the right (see Figure 1).

VERTICAL AND HORIZONTAL SHIFTS Suppose $c > 0$. To obtain the graph of

$y = f(x) + c$, shift the graph of $y = f(x)$ a distance c units upward

$y = f(x) - c$, shift the graph of $y = f(x)$ a distance c units downward

$y = f(x - c)$, shift the graph of $y = f(x)$ a distance c units to the right

$y = f(x + c)$, shift the graph of $y = f(x)$ a distance c units to the left

EXAMPLE 9 Given $F(x) = \cos^2(x + 9)$, find functions f, g, and h such that $F = f \circ g \circ h$.

SOLUTION Since $F(x) = [\cos(x + 9)]^2$, the formula for F says: First add 9, then take the cosine of the result, and finally square. So we let

$$h(x) = x + 9 \qquad g(x) = \cos x \qquad f(x) = x^2$$

Then

$$\begin{aligned}(f \circ g \circ h)(x) &= f(g(h(x))) = f(g(x + 9)) = f(\cos(x + 9)) \\ &= [\cos(x + 9)]^2 = F(x)\end{aligned}$$

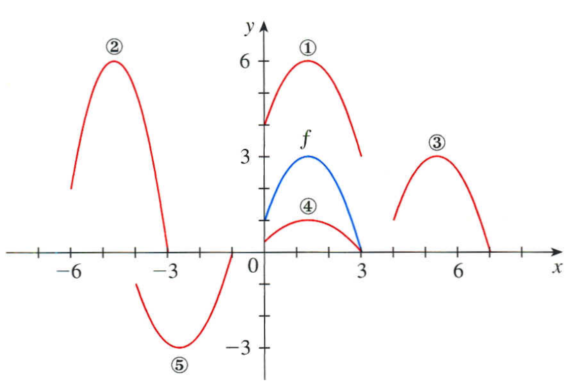
□

1.3 EXERCISES

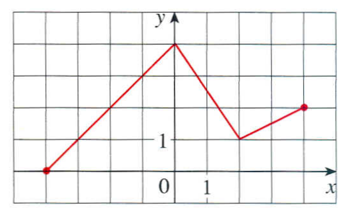
1. Suppose the graph of f is given. Write equations for the graphs that are obtained from the graph of f as follows.
(a) Shift 3 units upward.
(b) Shift 3 units downward.
(c) Shift 3 units to the right.
(d) Shift 3 units to the left.
(e) Reflect about the x-axis.
(f) Reflect about the y-axis.
(g) Stretch vertically by a factor of 3.
(h) Shrink vertically by a factor of 3.

2. Explain how each graph is obtained from the graph of $y = f(x)$.
(a) $y = 5f(x)$ (b) $y = f(x - 5)$
(c) $y = -f(x)$ (d) $y = -5f(x)$
(e) $y = f(5x)$ (f) $y = 5f(x) - 3$

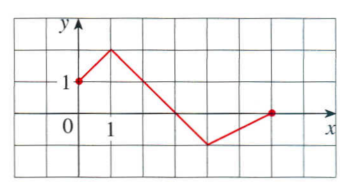
3. The graph of $y = f(x)$ is given. Match each equation with its graph and give reasons for your choices.
(a) $y = f(x - 4)$ (b) $y = f(x) + 3$
(c) $y = \frac{1}{3}f(x)$ (d) $y = -f(x + 4)$
(e) $y = 2f(x + 6)$



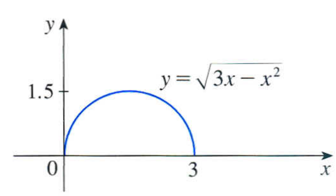
4. The graph of f is given. Draw the graphs of the following functions.
(a) $y = f(x + 4)$ (b) $y = f(x) + 4$
(c) $y = 2f(x)$ (d) $y = -\frac{1}{2}f(x) + 3$



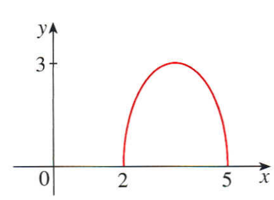
5. The graph of f is given. Use it to graph the following functions.
(a) $y = f(2x)$ (b) $y = f(\frac{1}{2}x)$
(c) $y = f(-x)$ (d) $y = -f(-x)$



6–7 The graph of $y = \sqrt{3x - x^2}$ is given. Use transformations to create a function whose graph is as shown.



6.



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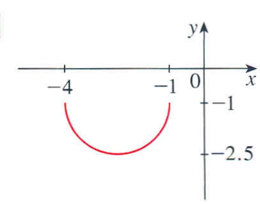


8. (a) How is the graph of $y = 2 \sin x$ related to the graph of $y = \sin x$? Use your answer and Figure 6 to sketch the graph of $y = 2 \sin x$.
(b) How is the graph of $y = 1 + \sqrt{x}$ related to the graph of $y = \sqrt{x}$? Use your answer and Figure 4(a) to sketch the graph of $y = 1 + \sqrt{x}$.

9–24 Graph the function by hand, not by plotting points, but by starting with the graph of one of the standard functions given in Section 1.2, and then applying the appropriate transformations.

9. $y = -x^3$

10. $y = 1 - x^2$

11. $y = (x + 1)^2$

12. $y = x^2 - 4x + 3$

13. $y = 1 + 2 \cos x$

14. $y = 4 \sin 3x$

15. $y = \sin(x/2)$

16. $y = \dfrac{1}{x - 4}$

17. $y = \sqrt{x + 3}$

18. $y = (x + 2)^4 + 3$

19. $y = \frac{1}{2}(x^2 + 8x)$

20. $y = 1 + \sqrt[3]{x - 1}$

21. $y = \dfrac{2}{x + 1}$

22. $y = \dfrac{1}{4} \tan\left(x - \dfrac{\pi}{4}\right)$

23. $y = |\sin x|$

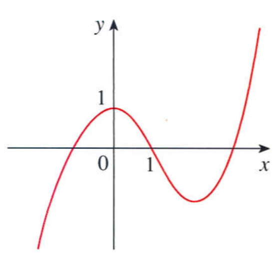
24. $y = |x^2 - 2x|$

25. The city of New Delhi, India, is located near latitude 30°N. Use Figure 9 to find a function that models the number of hours of daylight at New Delhi as a function of the time of year. To check the accuracy of your model, use the fact that on March 31 the sun rises at 6:13 AM and sets at 6:39 PM in New Delhi.

26. A variable star is one whose brightness alternately increases and decreases. For the most visible variable star, Delta Cephei, the time between periods of maximum brightness is 5.4 days, the average brightness (or magnitude) of the star is 4.0, and its brightness varies by ± 0.35 magnitude. Find a function that models the brightness of Delta Cephei as a function of time.

27. (a) How is the graph of $y = f(|x|)$ related to the graph of f?
(b) Sketch the graph of $y = \sin |x|$.
(c) Sketch the graph of $y = \sqrt{|x|}$.

28. Use the given graph of f to sketch the graph of $y = 1/f(x)$. Which features of f are the most important in sketching $y = 1/f(x)$? Explain how they are used.



29–30 Find $f + g$, $f - g$, fg, and f/g and state their domains.

29. $f(x) = x^3 + 2x^2$, $g(x) = 3x^2 - 1$

30. $f(x) = \sqrt{3 - x}$, $g(x) = \sqrt{x^2 - 1}$

31–36 Find the functions (a) $f \circ g$, (b) $g \circ f$, (c) $f \circ f$, and (d) $g \circ g$ and their domains.

31. $f(x) = x^2 - 1$, $g(x) = 2x + 1$

32. $f(x) = x - 2$, $g(x) = x^2 + 3x + 4$

33. $f(x) = 1 - 3x$, $g(x) = \cos x$

34. $f(x) = \sqrt{x}$, $g(x) = \sqrt[3]{1 - x}$

35. $f(x) = x + \dfrac{1}{x}$, $g(x) = \dfrac{x + 1}{x + 2}$

36. $f(x) = \dfrac{x}{1 + x}$, $g(x) = \sin 2x$

37–40 Find $f \circ g \circ h$.

37. $f(x) = x + 1$, $g(x) = 2x$, $h(x) = x - 1$

38. $f(x) = 2x - 1$, $g(x) = x^2$, $h(x) = 1 - x$

39. $f(x) = \sqrt{x - 3}$, $g(x) = x^2$, $h(x) = x^3 + 2$

40. $f(x) = \tan x$, $g(x) = \dfrac{x}{x - 1}$, $h(x) = \sqrt[3]{x}$

41–46 Express the function in the form $f \circ g$.

41. $F(x) = (x^2 + 1)^{10}$

42. $F(x) = \sin(\sqrt{x})$

43. $F(x) = \dfrac{\sqrt[3]{x}}{1 + \sqrt[3]{x}}$

44. $G(x) = \sqrt[3]{\dfrac{x}{1 + x}}$

45. $u(t) = \sqrt{\cos t}$

46. $u(t) = \dfrac{\tan t}{1 + \tan t}$

47–49 Express the function in the form $f \circ g \circ h$.

47. $H(x) = 1 - 3^{x^2}$

48. $H(x) = \sqrt[8]{2 + |x|}$

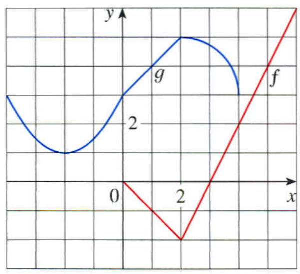
49. $H(x) = \sec^4(\sqrt{x})$

50. Use the table to evaluate each expression.
(a) $f(g(1))$ (b) $g(f(1))$ (c) $f(f(1))$
(d) $g(g(1))$ (e) $(g \circ f)(3)$ (f) $(f \circ g)(6)$

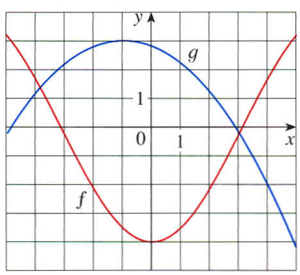
x	1	2	3	4	5	6
$f(x)$	3	1	4	2	2	5
$g(x)$	6	3	2	1	2	3

51. Use the given graphs of f and g to evaluate each expression, or explain why it is undefined.

(a) $f(g(2))$ (b) $g(f(0))$ (c) $(f \circ g)(0)$
(d) $(g \circ f)(6)$ (e) $(g \circ g)(-2)$ (f) $(f \circ f)(4)$



52. Use the given graphs of f and g to estimate the value of $f(g(x))$ for $x = -5, -4, -3, \ldots, 5$. Use these estimates to sketch a rough graph of $f \circ g$.



53. A stone is dropped into a lake, creating a circular ripple that travels outward at a speed of 60 cm/s.

(a) Express the radius r of this circle as a function of the time t (in seconds).
(b) If A is the area of this circle as a function of the radius, find $A \circ r$ and interpret it.

54. A spherical balloon is being inflated and the radius of the balloon is increasing at a rate of 2 cm/s.

(a) Express the radius r of the balloon as a function of the time t (in seconds).
(b) If V is the volume of the balloon as a function of the radius, find $V \circ r$ and interpret it.

55. A ship is moving at a speed of 30 km/h parallel to a straight shoreline. The ship is 6 km from shore and it passes a lighthouse at noon.

(a) Express the distance s between the lighthouse and the ship as a function of d, the distance the ship has traveled since noon; that is, find f so that $s = f(d)$.
(b) Express d as a function of t, the time elapsed since noon; that is, find g so that $d = g(t)$.
(c) Find $f \circ g$. What does this function represent?

56. An airplane is flying at a speed of 350 km/h at an altitude of 1 km and passes directly over a radar station at time $t = 0$.

(a) Express the horizontal distance d (in kilometers) that the plane has flown as a function of t.
(b) Express the distance s between the plane and the radar station as a function of d.
(c) Use composition to express s as a function of t.

57. The **Heaviside function** H is defined by

$$H(t) = \begin{cases} 0 & \text{if } t < 0 \\ 1 & \text{if } t \geqslant 0 \end{cases}$$

It is used in the study of electric circuits to represent the sudden surge of electric current, or voltage, when a switch is instantaneously turned on.

(a) Sketch the graph of the Heaviside function.
(b) Sketch the graph of the voltage $V(t)$ in a circuit if the switch is turned on at time $t = 0$ and 120 volts are applied instantaneously to the circuit. Write a formula for $V(t)$ in terms of $H(t)$.
(c) Sketch the graph of the voltage $V(t)$ in a circuit if the switch is turned on at time $t = 5$ seconds and 240 volts are applied instantaneously to the circuit. Write a formula for $V(t)$ in terms of $H(t)$. (Note that starting at $t = 5$ corresponds to a translation.)

58. The Heaviside function defined in Exercise 57 can also be used to define the **ramp function** $y = ctH(t)$, which represents a gradual increase in voltage or current in a circuit.

(a) Sketch the graph of the ramp function $y = tH(t)$.
(b) Sketch the graph of the voltage $V(t)$ in a circuit if the switch is turned on at time $t = 0$ and the voltage is gradually increased to 120 volts over a 60-second time interval. Write a formula for $V(t)$ in terms of $H(t)$ for $t \leqslant 60$.
(c) Sketch the graph of the voltage $V(t)$ in a circuit if the switch is turned on at time $t = 7$ seconds and the voltage is gradually increased to 100 volts over a period of 25 seconds. Write a formula for $V(t)$ in terms of $H(t)$ for $t \leqslant 32$.

59. Let f and g be linear functions with equations $f(x) = m_1x + b_1$ and $g(x) = m_2x + b_2$. Is $f \circ g$ also a linear function? If so, what is the slope of its graph?

60. If you invest x dollars at 4% interest compounded annually, then the amount $A(x)$ of the investment after one year is $A(x) = 1.04x$. Find $A \circ A$, $A \circ A \circ A$, and $A \circ A \circ A \circ A$. What do these compositions represent? Find a formula for the composition of n copies of A.

61. (a) If $g(x) = 2x + 1$ and $h(x) = 4x^2 + 4x + 7$, find a function f such that $f \circ g = h$. (Think about what operations you would have to perform on the formula for g to end up with the formula for h.)
(b) If $f(x) = 3x + 5$ and $h(x) = 3x^2 + 3x + 2$, find a function g such that $f \circ g = h$.

62. If $f(x) = x + 4$ and $h(x) = 4x - 1$, find a function g such that $g \circ f = h$.

63. (a) Suppose f and g are even functions. What can you say about $f + g$ and fg?
(b) What if f and g are both odd?

64. Suppose f is even and g is odd. What can you say about fg?

65. Suppose g is an even function and let $h = f \circ g$. Is h always an even function?

66. Suppose g is an odd function and let $h = f \circ g$. Is h always an odd function? What if f is odd? What if f is even?

1.4 EXERCISES

1. Use a graphing calculator or computer to determine which of the given viewing rectangles produces the most appropriate graph of the function $f(x) = \sqrt{x^3 - 5x^2}$.
(a) $[-5, 5]$ by $[-5, 5]$ (b) $[0, 10]$ by $[0, 2]$
(c) $[0, 10]$ by $[0, 10]$

2. Use a graphing calculator or computer to determine which of the given viewing rectangles produces the most appropriate graph of the function $f(x) = x^4 - 16x^2 + 20$.
(a) $[-3, 3]$ by $[-3, 3]$ (b) $[-10, 10]$ by $[-10, 10]$
(c) $[-50, 50]$ by $[-50, 50]$ (d) $[-5, 5]$ by $[-50, 50]$

3–14 Determine an appropriate viewing rectangle for the given function and use it to draw the graph.

3. $f(x) = 5 + 20x - x^2$
4. $f(x) = x^3 + 30x^2 + 200x$
5. $f(x) = \sqrt[4]{81 - x^4}$
6. $f(x) = \sqrt{0.1x + 20}$
7. $f(x) = x^3 - 225x$
8. $f(x) = \dfrac{x}{x^2 + 100}$
9. $f(x) = \sin^2(1000x)$
10. $f(x) = \cos(0.001x)$
11. $f(x) = \sin\sqrt{x}$
12. $f(x) = \sec(20\pi x)$
13. $y = 10\sin x + \sin 100x$
14. $y = x^2 + 0.02\sin 50x$

15. Graph the ellipse $4x^2 + 2y^2 = 1$ by graphing the functions whose graphs are the upper and lower halves of the ellipse.

16. Graph the hyperbola $y^2 - 9x^2 = 1$ by graphing the functions whose graphs are the upper and lower branches of the hyperbola.

17–18 Do the graphs intersect in the given viewing rectangle? If they do, how many points of intersection are there?

17. $y = 3x^2 - 6x + 1$, $y = 0.23x - 2.25$; $[-1, 3]$ by $[-2.5, 1.5]$

18. $y = 6 - 4x - x^2$, $y = 3x + 18$; $[-6, 2]$ by $[-5, 20]$

19–21 Find all solutions of the equation correct to two decimal places.

19. $x^3 - 9x^2 - 4 = 0$
20. $x^3 = 4x - 1$
21. $x^2 = \sin x$

22. We saw in Example 9 that the equation $\cos x = x$ has exactly one solution.
(a) Use a graph to show that the equation $\cos x = 0.3x$ has three solutions and find their values correct to two decimal places.
(b) Find an approximate value of m such that the equation $\cos x = mx$ has exactly two solutions.

23. Use graphs to determine which of the functions $f(x) = 10x^2$ and $g(x) = x^3/10$ is eventually larger (that is, larger when x is very large).

24. Use graphs to determine which of the functions $f(x) = x^4 - 100x^3$ and $g(x) = x^3$ is eventually larger.

25. For what values of x is it true that $|\sin x - x| < 0.1$?

26. Graph the polynomials $P(x) = 3x^5 - 5x^3 + 2x$ and $Q(x) = 3x^5$ on the same screen, first using the viewing rectangle $[-2, 2]$ by $[-2, 2]$ and then changing to $[-10, 10]$ by $[-10{,}000, 10{,}000]$. What do you observe from these graphs?

27. In this exercise we consider the family of root functions $f(x) = \sqrt[n]{x}$, where n is a positive integer.
(a) Graph the functions $y = \sqrt{x}$, $y = \sqrt[4]{x}$, and $y = \sqrt[6]{x}$ on the same screen using the viewing rectangle $[-1, 4]$ by $[-1, 3]$.
(b) Graph the functions $y = x$, $y = \sqrt[3]{x}$, and $y = \sqrt[5]{x}$ on the same screen using the viewing rectangle $[-3, 3]$ by $[-2, 2]$. (See Example 7.)
(c) Graph the functions $y = \sqrt{x}$, $y = \sqrt[3]{x}$, $y = \sqrt[4]{x}$, and $y = \sqrt[5]{x}$ on the same screen using the viewing rectangle $[-1, 3]$ by $[-1, 2]$.
(d) What conclusions can you make from these graphs?

28. In this exercise we consider the family of functions $f(x) = 1/x^n$, where n is a positive integer.
(a) Graph the functions $y = 1/x$ and $y = 1/x^3$ on the same screen using the viewing rectangle $[-3, 3]$ by $[-3, 3]$.
(b) Graph the functions $y = 1/x^2$ and $y = 1/x^4$ on the same screen using the same viewing rectangle as in part (a).
(c) Graph all of the functions in parts (a) and (b) on the same screen using the viewing rectangle $[-1, 3]$ by $[-1, 3]$.
(d) What conclusions can you make from these graphs?

29. Graph the function $f(x) = x^4 + cx^2 + x$ for several values of c. How does the graph change when c changes?

30. Graph the function $f(x) = \sqrt{1 + cx^2}$ for various values of c. Describe how changing the value of c affects the graph.

31. Graph the function $y = x^n 2^{-x}$, $x \geq 0$, for $n = 1, 2, 3, 4, 5,$ and 6. How does the graph change as n increases?

32. The curves with equations

$$y = \frac{|x|}{\sqrt{c - x^2}}$$

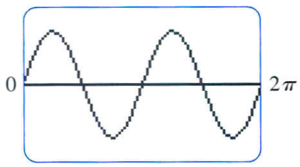
are called **bullet-nose curves**. Graph some of these curves to see why. What happens as c increases?

33. What happens to the graph of the equation $y^2 = cx^3 + x^2$ as c varies?

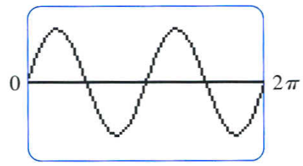
34. This exercise explores the effect of the inner function g on a composite function $y = f(g(x))$.
(a) Graph the function $y = \sin(\sqrt{x})$ using the viewing rectangle $[0, 400]$ by $[-1.5, 1.5]$. How does this graph differ from the graph of the sine function?

(b) Graph the function $y = \sin(x^2)$ using the viewing rectangle $[-5, 5]$ by $[-1.5, 1.5]$. How does this graph differ from the graph of the sine function?

35. The figure shows the graphs of $y = \sin 96x$ and $y = \sin 2x$ as displayed by a TI-83 graphing calculator.



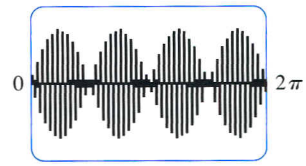
$y = \sin 96x$

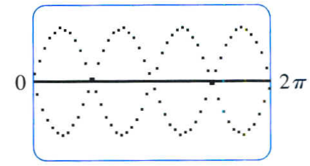


$y = \sin 2x$

The first graph is inaccurate. Explain why the two graphs appear identical. [*Hint:* The TI-83's graphing window is 95 pixels wide. What specific points does the calculator plot?]

36. The first graph in the figure is that of $y = \sin 45x$ as displayed by a TI-83 graphing calculator. It is inaccurate and so, to help explain its appearance, we replot the curve in dot mode in the second graph.





What two sine curves does the calculator appear to be plotting? Show that each point on the graph of $y = \sin 45x$ that the TI-83 chooses to plot is in fact on one of these two curves. (The TI-83's graphing window is 95 pixels wide.)

1 REVIEW

CONCEPT CHECK

1. (a) What is a function? What are its domain and range?
   (b) What is the graph of a function?
   (c) How can you tell whether a given curve is the graph of a function?

2. Discuss four ways of representing a function. Illustrate your discussion with examples.

3. (a) What is an even function? How can you tell if a function is even by looking at its graph?
   (b) What is an odd function? How can you tell if a function is odd by looking at its graph?

4. What is an increasing function?

5. What is a mathematical model?

6. Give an example of each type of function.
   (a) Linear function (b) Power function
   (c) Exponential function (d) Quadratic function
   (e) Polynomial of degree 5 (f) Rational function

7. Sketch by hand, on the same axes, the graphs of the following functions.
   (a) $f(x) = x$ (b) $g(x) = x^2$
   (c) $h(x) = x^3$ (d) $j(x) = x^4$

8. Draw, by hand, a rough sketch of the graph of each function.
   (a) $y = \sin x$ (b) $y = \tan x$
   (c) $y = 2^x$ (d) $y = 1/x$
   (e) $y = |x|$ (f) $y = \sqrt{x}$

9. Suppose that f has domain A and g has domain B.
   (a) What is the domain of $f + g$?
   (b) What is the domain of fg?
   (c) What is the domain of f/g?

10. How is the composite function $f \circ g$ defined? What is its domain?

11. Suppose the graph of f is given. Write an equation for each of the graphs that are obtained from the graph of f as follows.
   (a) Shift 2 units upward.
   (b) Shift 2 units downward.
   (c) Shift 2 units to the right.
   (d) Shift 2 units to the left.
   (e) Reflect about the x-axis.
   (f) Reflect about the y-axis.
   (g) Stretch vertically by a factor of 2.
   (h) Shrink vertically by a factor of 2.
   (i) Stretch horizontally by a factor of 2.
   (j) Shrink horizontally by a factor of 2.

TRUE-FALSE QUIZ

Determine whether the statement is true or false. If it is true, explain why. If it is false, explain why or give an example that disproves the statement.

1. If f is a function, then $f(s + t) = f(s) + f(t)$.

2. If $f(s) = f(t)$, then $s = t$.

3. If f is a function, then $f(3x) = 3f(x)$.

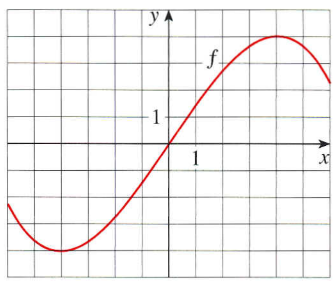
4. If $x_1 < x_2$ and f is a decreasing function, then $f(x_1) > f(x_2)$.

5. A vertical line intersects the graph of a function at most once.

6. If f and g are functions, then $f \circ g = g \circ f$.

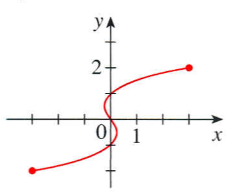
EXERCISES

1. Let f be the function whose graph is given.
(a) Estimate the value of $f(2)$.
(b) Estimate the values of x such that $f(x) = 3$.
(c) State the domain of f.
(d) State the range of f.
(e) On what interval is f increasing?
(f) Is f even, odd, or neither even nor odd? Explain.

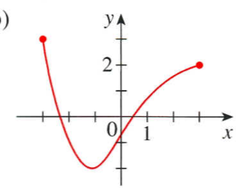


2. Determine whether each curve is the graph of a function of x. If it is, state the domain and range of the function.

(a)



(b)



3. If $f(x) = x^2 - 2x + 3$, evaluate the difference quotient

$$\frac{f(a+h) - f(a)}{h}$$

4. Sketch a rough graph of the yield of a crop as a function of the amount of fertilizer used.

5–8 Find the domain and range of the function.

5. $f(x) = 2/(3x - 1)$

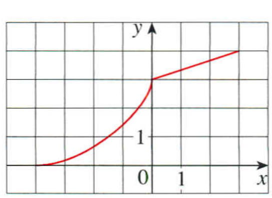
6. $g(x) = \sqrt{16 - x^4}$

7. $y = 1 + \sin x$

8. $F(t) = 3 + \cos 2t$

9. Suppose that the graph of f is given. Describe how the graphs of the following functions can be obtained from the graph of f.
(a) $y = f(x) + 8$
(b) $y = f(x + 8)$
(c) $y = 1 + 2f(x)$
(d) $y = f(x - 2) - 2$
(e) $y = -f(x)$
(f) $y = 3 - f(x)$

10. The graph of f is given. Draw the graphs of the following functions.



(a) $y = f(x - 8)$
(b) $y = -f(x)$
(c) $y = 2 - f(x)$
(d) $y = \frac{1}{2}f(x) - 1$

11–16 Use transformations to sketch the graph of the function.

11. $y = -\sin 2x$

12. $y = (x - 2)^2$

13. $y = 1 + \frac{1}{2}x^3$

14. $y = 2 - \sqrt{x}$

15. $f(x) = \dfrac{1}{x + 2}$

16. $f(x) = \begin{cases} 1 + x & \text{if } x < 0 \\ 1 + x^2 & \text{if } x \geqslant 0 \end{cases}$

17. Determine whether f is even, odd, or neither even nor odd.
(a) $f(x) = 2x^5 - 3x^2 + 2$
(b) $f(x) = x^3 - x^7$
(c) $f(x) = \cos(x^2)$
(d) $f(x) = 1 + \sin x$

18. Find an expression for the function whose graph consists of the line segment from the point $(-2, 2)$ to the point $(-1, 0)$ together with the top half of the circle with center the origin and radius 1.

19. If $f(x) = \sqrt{x}$ and $g(x) = \sin x$, find the functions (a) $f \circ g$, (b) $g \circ f$, (c) $f \circ f$, (d) $g \circ g$, and their domains.

20. Express the function $F(x) = 1/\sqrt{x + \sqrt{x}}$ as a composition of three functions.

21. Use graphs to discover what members of the family of functions $f(x) = \sin^n x$ have in common, where n is a positive integer. How do they differ? What happens to the graphs as n becomes large?

22. A small-appliance manufacturer finds that it costs \$9000 to produce 1000 toaster ovens a week and \$12,000 to produce 1500 toaster ovens a week.
(a) Express the cost as a function of the number of toaster ovens produced, assuming that it is linear. Then sketch the graph.
(b) What is the slope of the graph and what does it represent?
(c) What is the y-intercept of the graph and what does it represent?

23. The table gives the population of Indonesia (in millions) for the years 1950–2000. Decide what type of model is appropriate and use your model to predict the population of Indonesia in 2010.

Year	Population	Year	Population
1950	80	1980	150
1955	86	1985	166
1960	96	1990	182
1965	107	1995	197
1970	120	2000	212
1975	134		

Then $$f_{k+1}(x) = (f_0 \circ f_k)(x) = f_0(f_k(x)) = f_0\left(\frac{x}{(k+1)x+1}\right)$$

$$= \frac{\dfrac{x}{(k+1)x+1}}{\dfrac{x}{(k+1)x+1}+1} = \frac{\dfrac{x}{(k+1)x+1}}{\dfrac{(k+2)x+1}{(k+1)x+1}} = \frac{x}{(k+2)x+1}$$

This expression shows that (4) is true for $n = k + 1$. Therefore, by mathematical induction, it is true for all positive integers n. □

PROBLEMS

1. One of the legs of a right triangle has length 4 cm. Express the length of the altitude perpendicular to the hypotenuse as a function of the length of the hypotenuse.

2. The altitude perpendicular to the hypotenuse of a right triangle is 12 cm. Express the length of the hypotenuse as a function of the perimeter.

3. Solve the equation $|2x - 1| - |x + 5| = 3$.

4. Solve the inequality $|x - 1| - |x - 3| \geqslant 5$.

5. Sketch the graph of the function $f(x) = |x^2 - 4|x| + 3|$.

6. Sketch the graph of the function $g(x) = |x^2 - 1| - |x^2 - 4|$.

7. Draw the graph of the equation $x + |x| = y + |y|$.

8. Draw the graph of the equation $x^4 - 4x^2 - x^2y^2 + 4y^2 = 0$.

9. Sketch the region in the plane consisting of all points (x, y) such that $|x| + |y| \leqslant 1$.

10. Sketch the region in the plane consisting of all points (x, y) such that

$$|x - y| + |x| - |y| \leqslant 2$$

11. A driver sets out on a journey. For the first half of the distance she drives at the leisurely pace of 60 km/h; she drives the second half at 120 km/h. What is her average speed on this trip?

12. Is it true that $f \circ (g + h) = f \circ g + f \circ h$?

13. Prove that if n is a positive integer, then $7^n - 1$ is divisible by 6.

14. Prove that $1 + 3 + 5 + \cdots + (2n - 1) = n^2$.

15. If $f_0(x) = x^2$ and $f_{n+1}(x) = f_0(f_n(x))$ for $n = 0, 1, 2, \ldots$, find a formula for $f_n(x)$.

16. (a) If $f_0(x) = \dfrac{1}{2 - x}$ and $f_{n+1} = f_0 \circ f_n$ for $n = 0, 1, 2, \ldots$, find an expression for $f_n(x)$ and use mathematical induction to prove it.

(b) Graph f_0, f_1, f_2, f_3 on the same screen and describe the effects of repeated composition.