中央研究院數學研究所
Institute of Mathematics
Academia Sinica

國立台灣大學數學系
Department of Mathematics
National Taiwan University

Lakeside Lectures

Speaker: Prof. Wen-Ching Winnie Li
(The Pennsylvania State University)

Title: Various aspects of hypergeometric functions

Abstract: The Gauss hypergeometric function ${}_2F_1(a, b; c; z)$ is a special function expressed by the hypergeometric series that includes many other special functions as specific or limiting cases. It satisfies a second order linear ODE, denoted $DE(a, b; c)$ with three regular singularities at $0, 1, \infty$. The solutions to $DE(a, b; c)$ form a rank-2 sheaf on $\mathbb{P}^1(\mathbb{C}) \setminus \{0, 1, \infty\}$, admitting the action of the fundamental group $\pi_1(\mathbb{P}^1(\mathbb{C}) \setminus \{0, 1, \infty\}, *)$, called the monodromy representation associated to ${}_2F_1(a, b; c; z)$.
The above generalizes from ${}_2F_1$ to ${}_nF_{n-1}$ when the set of parameters $\{a, b; c\}$ is extended to a hypergeometric datum $HD = \{a_1, \cdots, a_n; b_1 = 1, b_2, \cdots, b_n\}$ with a_i, b_j nonzero rationals. A parallel algebraic setting was introduced by Katz who defined a hypergeometric sheaf $\mathcal{F}(HD)$ on the multiplicative (algebraic) group $\mathbf{G}_m$ together with a representation of the absolute Galois group of a cyclotomic field acting on $\mathcal{F}(HD)$. A well-known Whipple identity, proved analytically, relates ${}_7F_6$ to ${}_4F_3$. We shall reinterpret it in terms of the associated Katz representations. Moreover, these Galois representations, arising geometrically, are shown to be automorphic, as predicted by the Langlands program. Finally, we shall discuss the connection between special values of hypergeometric functions and periods. This is a joint work with Ling Long and Fang-Ting Tu.

Time: 14:00 – 15:00, Monday, May 29, 2023
Venue: Room 202, Astro-Math. Building (NTU Campus)
Refreshment: 15:00
Organizers: Ching Wei Ho, Chun-Ju Lai, Wai Kit Lam, Cheng-Chiang Tsai