特殊選才：線性代數

109 學年度

(1) Denote by P the set of all single-variable (say, in x) polynomials with real coefficients, and of degree no greater than 3. P naturally carries a vector space structure (over $\mathbb{R}$).

For each of the following subsets, determine whether it is a vector subspace of P or not. If YES, find out its dimension, and give a basis.

中文版：考慮所有三次以下的實係數多項式，將此集合記為 P。P 自然是一個向量空間。

$$P = \{\, f(x) \text{ 是一個多項式} \mid \deg f(x) \leq 3 \,\}\,.$$

請問下面的子集合是否為向量子空間。若是，請求出其維度、並找出一組基底。

(a) $U = \{f(x) \in P \mid f(0) = 3\}$.
(b) $V = \{f(x) \in P \mid f(1) = f(-1)\}$.
(c) $W = \{f(x) \in P \mid f(2) = 0\}$.