臺灣大學數學系
99學年度上學期博士班資格考試題
科目：統計

2010.09.17

1. Let $U_1, \cdots, U_n, \cdots$ be *i.i.d.* $U(0,1)$ random variables, X have the p.d.f. $P(X = x) = 1/((e-1)x!)I_{\{1,2,\cdots\}}(x)$, and $S_n = \sum_{i=1}^{n} U_i$.

(1a) (10%) Derive the distribution of $Z = \min\{U_1, \cdots, U_X\}$.

(1b) (10%) Compute the expectaqtion of $N = \min\{n : S_n > 1\}$.

2. Let $X_1, \cdots, X_n$ be a random sample from a p.d.f $f(x)$ with median M.

(2a) (10%) Suppose that $f(M) \neq 0$. Derive the limiting distribution of $\sqrt{n}(M_n - M)$, where $M = Median\{X_1, \cdots, X_n\}$.

(2b) (10%) Suppose that $g^{(1)}(x)$ is continuous at M and $g^{(1)}(M) \neq 0$. Find an approximated probability of $P(g(M_n) \leq x)$.

3. Let $X_1, \cdots, X_n$ be a random sample from $Bernoulli(\pi)$ with $n > 2$.

(3a) (10%) Find the uniformly minimum variance unbiased estimator of π^2.

(3b) (4%) (6%) Find the maximum likelihood estimator of π^2 and show that the one-step jackknife estimator is an unbiased estimator of π^2.

4. (20%) Let $X_1, \cdots, X_n$ be a random sample from a p.d.f $f(x|\theta)$ with $\theta \in \Theta$ and $dim(\Theta) = k$. Consider the hypotheses $H_0 : \theta \in \Theta_0$ versus $H_A : \theta \in \Theta - \Theta_0$, where $\Theta_0 = \{\theta : \theta = g(\eta)\} \subset \Theta$ with η being a $(k-r) \times 1$ unknown parameter vector and $g(\cdot)$ being a continuously differential function from R^{k-r} to R^k. Show that $-2\ln(\lambda_n) \xrightarrow{d} \chi_r^2$ under H_0 and the regularity conditions, where λ_n is the likelihood ratio test statistic.

5. (10%) Show that a uniformly most powerful test is an unbiased test.

6. (10%) Let $\widehat{\delta}$ be the Bayes estimator with constant risk. Show that $\widehat{\delta}$ be the minimax estimator.