臺灣大學數學系
100 學年度下學期博士班資格考試題
科目：統計 2012.02.23

1. (7%) (8%) Let $X|Y = y \sim Binomial(y, p)$, $Y|\Lambda = \lambda \sim Poisson(\lambda)$, and $\Lambda \sim Exponential(\beta)$. Compute the expectation and variance of X.

2. (15%) Let $X_1, \ldots, X_n$ be a random sample from $Uniform(\alpha - \beta, \alpha + \beta)$ where α and β are unknown parameters. Find the uniformly minimum variance unbiased estimator of α/β.

3. (10%) Let $X_1, \ldots, X_n$ be a random sample from $Poisson(\lambda)$ and λ have a $Gamma(\alpha, \beta)$ distribution, where α and β are known positive constants. Find the Bayes estimator of λ under the squared error loss function.

4. (10%) (10%) Let $X_1, \ldots, X_n$ be a random sample from $f(x|\theta)$, where θ is a real-valued parameter. For each $\theta_0 \in \Theta$, consider the null hypothesis $H_0 : \theta = \theta_0$ versus the alternative hypothesis $H_A : \theta \in \Theta - \{\theta_0\}$. Show that a non-randomized uniformly most powerful level α test exists if and only if a $(1 - \alpha)$ uniformly most accurate confidence set for θ exists.

5. (15%) Let X follow a p-variate spherical distribution, i.e. the density $f(x)$ of X depends on x only through $x^\top x$. Show that the characteristic function $\phi(t) = E[\exp(it^\top X)]$ is a function of $t^\top t$.

6. (10%) (15%) Let $F_T(t)$ and $F_C(c)$ denote separately the cumulative distribution functions of the non-negative continuous random variables T and C. Suppose that T and C are independent and C is noninformative. Based on a random sample of the form $\{(X_i, \delta_i)\}_{i=1}^n$, where $X_i = \min\{T_i, C_i\}$ and $\delta_i = I(X_i = T_i)$, write down the corresponding likelihood function and derive the the maximum likelihood estimator of $F_T(t)$.