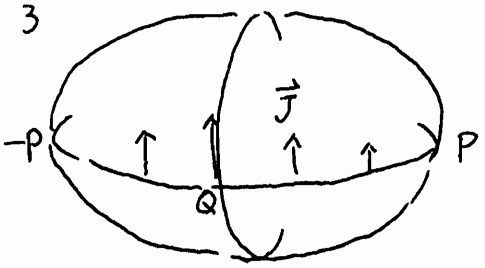
臺灣大學數學系
九十七學年度下學期博士班資格考試題
科目：幾何與拓樸　　　　2009.02.26

1. $\omega = (x^2-y^2-z^2)\,dx + 2xy\,dy + 2xz\,dz$ is a differential 1-form. Is it a closed 1-form? If not, can you find an integrating factor $\mu = \mu(x,y,z)$ so that $\mu\cdot\omega = df$ is an exact 1-form? If yes, $f = ? + C$ (25/100)

2. Let M be a closed surface, so M is a compact 2-dimensional manifold without boundary. $f : M \to M$ is a continuous map. If M is simply connected, can you prove by Brouwer fixed point theorem or any other method, that there must be at least one fixed point $x \in M$ satisfying $f(x) = x$? (25/100)

3



$\frac{x^2}{9} + \frac{y^2}{4} + \frac{z^2}{1} = 1$ is an ellipsoid.

$P = (3,0,0)$, $Q = (0,2,0)$, $-P = (-3,0,0)$

The arc $\Gamma = \widehat{-P,Q,P}$ on the x-y plane is a geodesic. Are the endpoints P & $-P$ conjugate to each other along Γ? If yes, can you find a Jacobi field $\vec{J} = (0,0,j_3(x))$ along Γ so that $j_3(x=-3) = 0 = j_3(x=+3)$? $j_3(x) = ?$ (25/100)

4. Can you find an umbilic point on the ellipsoid $x^2/9 + y^2/4 + z^2 = 1$ where all the normal curvatures in each tangential direction are the same? If yes $(x,y,z) = (?,?,?)$ (25/100)