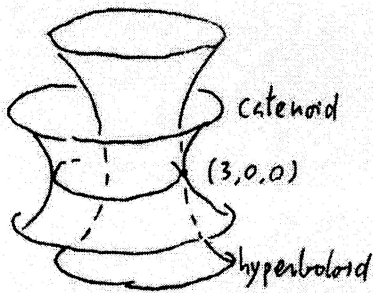


國立臺灣大學數學系  
九十五學年度博士班資格考試試題  
科目：幾何與拓樸

2007.06.01



(25/100)

1. Can you find a catenoid

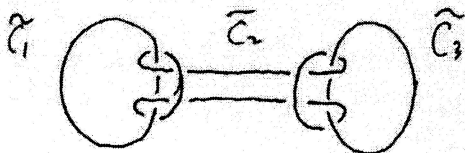
$\lambda \sqrt{(x-a)^2 + (y-b)^2} = \cosh \lambda z$  outside of the hyperboloid so that  $x^2/9 + y^2/4 - z^2/1 \neq 1$  except the point  $(3, 0, 0)$  where they intersect? If yes,  $a = ?$ ,  $b = ?$ ,  $\lambda = ?$

2.  $\frac{x^2}{16} + \frac{y^2}{9} + \frac{z^2}{4} - w^2 = 1$  is a 3-manifold in  $\mathbb{R}^4$ ,

$ds^2 = dx^2 + dy^2 + dz^2 + dw^2$ . Is  $(0, 3, 0, 0)$  a point of positive Ricci curvature? If not, can you conclude it is a point of negative Ricci curvature? (25/100)

3. Let  $C_1, C_2, C_3$  be three circles on the plane  $z=0$

$C_1 = \{x^2 + y^2 - 1 = 0 = z\}$ ,  $C_2 = \{(x-3)^2 + y^2 - 1 = 0 = z\}$ ,  $C_3 = \{(x+3)^2 + y^2 - 1 = 0 = z\}$ . Find the fundamental group  $\pi_1(\mathbb{R}^3 - C_1 - C_2 - C_3) = ?$



If  $\tilde{C}_1, \tilde{C}_2, \tilde{C}_3$  are linked like this,  $\pi_1(\mathbb{R}^3 - \tilde{C}_1 - \tilde{C}_2 - \tilde{C}_3) = ?$

(25/100)

4.  $\{z = x + iy \mid y > 0\}$  is the upper half plane,  $ds^2 = \frac{dx^2 + dy^2}{y^2}$  is the Poincaré metric.  $\gamma$  is an arc from  $i$  to  $2i$ .

$\int_{\gamma} ds = ?$   $W = \frac{az+b}{cz+d}$   $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2, \mathbb{R})$  is a

Möbius transform, does it preserve  $ds^2$ ? If  $\Gamma$  is

an arc from  $i$  to  $i+1$ ,  $\int_{\Gamma} ds = ?$  (25/100)