

臺灣大學數學系  
99 學年度上學期博士班資格考試題  
科目：代數

2010.09.17

1. (30%)
  - (a) Classify simple groups of order 60 up to isomorphism.
  - (b) Classify groups of order 30 up to isomorphism.
2. (15%)
  - (a) Is  $\mathbf{Z}[\sqrt{-2}]$  a UFD?
  - (b) Is the polynomial  $x^4 - 4x^2 + 8x + 2$  irreducible over the quadratic field  $\mathbf{Q}(\sqrt{-2})$ ?  
(Justify your answers)
3. (25%) Let  $R$  be a PID.
  - (a) Show that  $M$  is a flat  $R$ -module if and only if  $M$  is a torsion free  $R$ -module.
  - (b) Let  $M$  be a finitely generated  $R$ -module. Show that  $M$  is free if and only if  $M$  is torsion free if and only if  $M$  is flat if and only if  $M$  is projective.
4. (30%) Compute the Galois groups over  $\mathbf{Q}$  of the following polynomials and then determine whether they are solvable by radicals.
  - (a)  $x^5 - 2$ .
  - (b)  $x^5 + 20x + 16$ .
  - (c)  $x^5 - 4x + 2$ .(Justify your answers)

(Here,  $\mathbf{Q}$  denotes the field of rational numbers and  $\mathbf{Z}$  denotes the ring of integers)