臺灣大學數學系
九十七學年度下學期博士班資格考試題
科目：代數　　2009.02.26

Do all the problems.

1. (10 pts.) Classify up to isomorphism all groups of order 33.

2. (20 pts.) Show that every finite multiplicative subgroups of a field is cyclic.

3. (15 pts.) Show that $\mathbb{C}[x,y,z]/(yz-x^2)$ is NOT isomorphic to $\mathbb{C}[s,t,u]/(u^2-s+t)$.

4. (15 pts.) Let F be an algebraically closed field. Show that an ideal $\mathfrak{m} \triangleleft F[x,y]$ is maximal if and only if $\mathfrak{m} = (x-a, y-b)$ for some $a, b \in F$.

5. (15 pts.) Let F/K be an algebraic extension over a field K of characteristic $p > 0$. An element $u \in F$ is said to be purely inseparable over K if there is an integer $n \geq 0$ such that $u^{p^n} \in K$.

   (a) Give a non-trivial example of purely inseparable extension.

   (b) Let L be the subset of F of all purely inseparable elements over K. Show that L is a field.

   (c) Keep the notation as above. Is F separable over L? Verify your answer.

6. (15 pts.) Construct a polynomial in $\mathbb{Q}[x]$ such that its Galois group over $\mathbb{Q}$ is S_5.

7. (10 pts.) Let $A \in M(4, \mathbb{R})$ be a 4×4 matrix such that $A^2 = 4A$. Find all possible Jordan forms of A.