台灣大學數學系

九十三學年度博士班入學考試題

統計

June 4, 2004

(1)

A random variable Y has a Lognormal (μ, σ^2) distribution if $\log Y \sim$ Normal (μ, σ^2). Let $Y_i \sim$ Lognormal (μ_i, σ_i^2), $i = 1, \cdots, n$ be independent.

(a)

(10 points) Find the distribution and expected value of $\Pi_{i=1}^n Y_i$.

(b)

(5 points) Find the distribution of Y_1/Y_2.

(2)

Let $X_1, \cdots, X_n$ be independent Bernoulli (p), $0 < p < 1$.

(a)

(7 points) Find the uniformly minimum variance unbiased estimator of $p(1-p)$.

(b)

(14 points) Find a variance stabilizing transformation of the sample mean $\bar{X}$. Construct an approximate level-α confidence interval of p based on the variance stabilizing transformation.

(3)

Let $X_1, \cdots, X_n$, $n \geq 2$ be independent and identically distributed with density

$$f(x; \mu, \sigma) = \frac{1}{\sigma} \exp\{-(x-\mu)/\sigma\} I_{[\mu,\infty)}(x),$$

where $-\infty < \mu < \infty$ and $\sigma > 0$ are both unknown.

(a)

(14 points) Show that $X_{(1)} = \min\{X_1, \cdots, X_n\}$ is complete and sufficient for μ for each fixed value of σ. Find the uniformly minimum variance unbiased estimator of μ.

(b)

(20 points) Suppose that $\mu \leq 0$. Find the maximum likelihood estimates of μ and σ and denote them as $\hat{\mu}_1$ and $\hat{\sigma}_1$. Find the large sample distribution of $\hat{\mu}_1$.

(c)

(10 points) Find the likelihood ratio test of $H_0 : \mu \leq 0$ versus $H_1 : \mu > 0$.

(4)

(20 points) Suppose $X_1, \cdots, X_n$ is a random sample from a discrete distribution with frequency function

$$f(x;\theta) = \frac{1}{\theta+1}\left(\frac{\theta}{\theta+1}\right)^x, \; x = 0,\ 1,\ 2\cdots$$

and zero otherwise, where $\theta > 0$. Find a uniformly most powerful test of $H_0 : \theta = \theta_0$ versus $H_1 : \theta > \theta_0$.