臺灣大學數學系

九十四學年度博士班入學考試題

迴歸分析

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Suppose that

$$Y_i = \beta_0 + \beta_1 X_{i1} + \cdots + \beta_k X_{ik} + \varepsilon_i,\ i = 1, \cdots, n,$$

where $\beta = (\beta_0, \beta_1, \cdots, \beta_k)'$ are unknown parameters, $X = \big(x_1, \cdots, x_n)'$ with $x_i = (1, X_{i1}, \cdots, X_{ik})'$ is a constant design matrix, and the errors $\varepsilon_1, \cdots, \varepsilon_n$ are independent with zero mean and variance σ^2. Assume that X is of full rank. The least squares estimate of β is $b = (X'X)^{-1}X'Y$, where $Y = (Y_1, \cdots, Y_n)'$.

1. In practice, it may occur that the error variances are unequal.
   (a) (8%) Describe a way to check unequal variance and a situation where unequal variances is expected.
   (b) (10%) Describe variance stabilizing transformation to correct for unequal variances, and discuss when it is appropriate.
   (c) (10%) Describe weighted least squares estimation to correct for unequal variances, and discuss when it is appropriate.

2. Consider ridge regression

$$\min_{\beta} \left\{(Y - X\beta)'(Y - X\beta) + k(\beta'\beta - c)\right\},$$

   where k and c are positive constants.
   (a) (5%) Find the solution of the ridge regression and denote it as b_k.
   (b) (8%) Find the mean and covariance of b_k.
   (c) (10%) Find the necessary and sufficient condition for b_k to have smaller mean squared error matrix than b.
   (d) (4%) Give one reason/interpretation for doing ridge regression.

3. Consider shrinkage estimator $b_\rho = (1 + \rho)^{-1}b$, where ρ is a positive constant.
   (a) (8%) Find the mean and covariance of b_ρ.
   (b) (8%) Find the necessary and sufficient condition for b_ρ to have smaller mean squared error matrix than b.
   (c) (4%) Give one reason/interpretation for doing shrinkage regression.

4. Consider the Box-Cox transformation of the response variable.

   (a) (5%) Write down the transformation.

   (b) (10%) Discuss purposes of the transformation.

   (c) (10%) Give a data adaptive procedure to determining the parameter in the transformation.