臺灣大學數學系

九十學年度博士班入學考試題

機率與統計

1.

(20 pts.) Let $(X_1, Y_1), \cdots, (X_n, Y_n)$ be a sample from a Bivariate $N(0, 0, \sigma_1^2, \sigma_2^2, \rho)$ distribution. Consider the problem of testing $H : \rho \neq 0$.

(a)

Show that the likelihood ratio statistic is equivalent to $|r|$, where

$$r = \frac{\sum_{i=1}^n X_i Y_i}{\sqrt{\sum i = 1^n X_i^2 \sum_{i=1}^n Y_i^2}}$$

(b)

Find the distribution of r^2 and construct a level 0.05 rejection region.

2.

(20pts.) Suppose that random variables $X_1, \cdots, X_n$ are serially correlated:

$$X_i = \theta X_{i-1} + \epsilon_i, \ i = 1, \cdots, n,$$

where $X_0 = 0$ and $\epsilon_1, \cdots, \epsilon_n$ are independent $N(0, \sigma^2)$ random variables.

(a)
Find the maximum likelihood estimates of θ and σ.
(b)
Construct a level α likelihood ratio test for $H : \theta = 0$ versus $K : \theta \neq 0$.

3.

(20 pts.) Let $X_i \sim$ Exponential (λ), for $i = 1, \cdots, m$ be independent.

(a)

Approximate the mean and variance of the kth order statistic $X_{(k)}$.

(a)

Approximate the mean and variance of the pth sample quantile.

4.

(20 pts.) Let $X_1, \cdots, X_n$ be i.i.d. from $Binomial(1, p)$.

(a)

Find an approximate level-α confidence interval for p using $\bar{X}$.

(b)

Find an approximate level-α confidence interval for p using a variance-stabilizing transformation of $\bar{X}$.

(c)

Compare the asymptotic length of the above two confidence intervals.

5.

(20 pts.) Show that $\bar{X}$ is minimax for the mean of normal distribution under quadratic loss.