台灣大學數學系

九十三學年度博士班入學考試題

機率論

June 4, 2004

(1)

(20 points) (a) Let $X_i, i = 1, 2, 3$ be three r.v.'s defined on a probability space $(\Omega, \mathcal{F}, P)$. If they are (totally) independent, prove that they are pairwise independent,i.e. any two of them are independent.

(b) Is the converse of (a) true? If it is true, give a proof; if it is not true, give a counter-example. pt

(2)

(20 pts) (a)Let X_n be a sequence of independent and identically distributed r.v.'s Assume that the 4th moment is finite, use Chebyshev's inequality to prove that SLLN for X_n. pt

(3)

.(20 pts) Let X have the geometric distribution with success probability $p \in (0, 1)$. Find the expectation, the variance and the moment generating function of X . pt

(4)

(20 pts) Let X_n be independent and have Poisson distribution with parameter 1. Let $S_n = \sum_{j=1}^{n} X_n$, Use CLT to prove that $(S_n - n)/\sqrt{n}$ converges in distribution to $N(0, 1)$. How about to repalce S_n by Z_λ where Z_λ has a Poisson distribution with mean λ. pt

(5)

(20 pts) Let X_n be a sequence of r.v.'s and $\mathcal{F}_n$ be an increasing sequence of sub-sigma algebras of $\mathcal{F}$. Assume that each X_n is a submartingale with respect to $\mathcal{F}_n$. (a) If ϕ is an increasing convex function with $E|\phi(X_n)| < \infty, \forall n$, prove that $\phi(X_n)$ is also a submartingale with respect to $\mathcal{F}_n$.

(b) Prove that $(X_n - a)^+$ is a submartingale if X_n is a submartingale, where a is a real.