台灣大學數學系

九十三學年度博士班入學考試題

數值PDE

June 4, 2004

(1)

State the finite difference theory for solving the Poisson equation on the square

$$\triangle\phi = f \quad \text{in } \Omega = [0,1]\times[0,1], \phi = 0 \quad \text{on } \partial\Omega,$$

where f is smooth. Your answer should include a second-order method, a pseudo-code, a brief description of the convergence theory, and an iterative method to solve the corresponding discretized linear equations. pt

(2)

This problem is an extension of the previous problem. It is to test how you think and how deep you can go in some research direction. You can answer whatever you want related to the issues.

(a)

Can you describe a numerical strategy for solving the Poisson equation on general domains?

(b)

If the boundary has corners, can you design a second-order scheme even around the corner points?

pt

(3)

Let us consider the linear advection equation in one dimension

$$u_t + au_x = 0, \; a > 0 \quad \text{is a constant.}$$

(a)

Design first-order and second-order schemes (in both space and time) for this equation.

(b)

Choose one scheme and analyze its stability.

(c)

State the Lax equivalence theorem.

pt

(4)

Let us partition the domain $[0,1]$ into N uniform subintervals. Let ϕ_i be the hat function at nodal point ih, $i = 1, ..., N-1$, where $h = 1/N$.

(a)

Using the above hat function, derive the finite element method for solving the equation

$$u_{xx} = f \quad \text{on } (0,1), \; u(0) = u(1) = 0.$$

(b)

Sketch the proof of the convergence of the finite element method for this case.

(5)

State anything you have learned about numerical PDEs which you think it is important. It should include a clear statement, why they are important, how they are used, and their potential of application potential. It should be more than one page.