臺灣大學數學系

九十五學年度博士班入學考試題

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1. Let $x = \rho \sin\phi \cos\theta$, $y = \rho \sin\phi \sin\theta$, $z = \rho \cos\phi$ be the Cartesian coordinates of the Euclidean space $\mathbb{R}^3$, and $\eta = \frac{1}{\rho^3}(xdy \wedge dz + ydz \wedge dx + zdx \wedge dy)$ be a differential 2-form on the open set $D = \{(x, y, z) \neq (0, 0, 0)\}$. Is η a closed 2-form? $\eta = d\omega$? If yes, $\omega =$? If you cannot find an ω on the whole domain D, let us restrict ourselves to $D^+ = \{(x, y, z) \in D |\ x > 0\}$. Can you find an anti-derivative ω in this smaller domain D^+? (25/100)

2. $S^3 = \{(x, y, z) \in \mathbb{R}^4 | x^2 + y^2 + z^2 + w^2 = 1\}$ is a 3-dimensional manifold with positive Ricci curvature. Can you find a different riemannian metric on S^3 so that at the point $(x, y, z, w) = (0, 0, 0, 1)$ the Ricci curvature is not non-negative $Ric(0, 0, 0, 1) \not\geq 0$, yet the Ricci curvature at $(0, 0, 0, 1)$ is not no-positive either, $Ric(0, 0, 0, 1) \not\leq 0$? (25/100)

3. Let $ds^2 = \frac{1}{y^2}(dx^2 + dy^2)$ be the Poincare metric on the upper half plane $H = \{y > 0\}$, and $\gamma = \{x^2 + y^2 = 2| -1 \leq x \leq 1\}$ be an arc from $p = (-1, 1)$ to $q = (1, 1)$. Let $\overrightarrow{u} = (1, 0)$ be a vector at p and $\overrightarrow{v}$ be the parallel translation of $\overrightarrow{u}$ from p to q along γ. $\overrightarrow{v} = (?, ?)$ (25/100)

4. Mean curvature is NOT an intrinsic quantity of surfaces in $\mathbb{R}^3$. Can you find an isometry map from the helicoid $H = \{\tan z = \frac{y}{x} \mid x^2 + y^2 < 1, 0 < z < \pi\}$ to another surface, which is not congruent to H, yet preserving the mean curvature of H? (25/100)