臺灣大學數學系

九十四學年度博士班入學考試題

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1. Let $U = \{(\theta, t) | t \geq 0\}$ be the upper half plane. Can you find smooth maps $x(\theta, t)$ and $y(\theta, t)$ periodic in θ with period 2π from U to $\mathbb{R}^2$ satisfying differential equations $\frac{\partial}{\partial t}(x, y) = \frac{\frac{\partial x}{\partial \theta}\frac{\partial^2 y}{\partial \theta^2} - \frac{\partial y}{\partial \theta}\frac{\partial^2 x}{\partial \theta^2}}{\left[(\frac{\partial x}{\partial \theta})^2 + (\frac{\partial y}{\partial \theta})^2\right]^{3/2}} \frac{\left(-\frac{\partial y}{\partial \theta}, \frac{\partial x}{\partial \theta}\right)}{\left[(\frac{\partial x}{\partial \theta})^2 + (\frac{\partial y}{\partial \theta})^2\right]^{1/2}}$? If not, can you find such maps from a subset of U to $\mathbb{R}^2$? (25/100)

2. $\omega = dx \wedge dy + dy \wedge dz + dz \wedge dw$ is a differential 2-form in $\mathbb{R}^4 = \{(x, y, z, w)\}$. Is ω a symplectic form? Can you find 1-forms α and β so that $\omega = \alpha \wedge \beta$? (25/100)

3. Let $C = \{z = 0 = x^2 + y^2 - 1\}$ be a circle in $\mathbb{R}^3$. Can you find a differential 1-form α on $\mathbb{R}^3 - C$ whic is closed but not exact? If yes, $\alpha = ?dx + ?dy + ?dz$ (25/100)

4. Can you find smooth maps $x(u, v)$, $y(u, v)$, $z(u, v)$ from $\mathbb{R}^2$to $\mathbb{R}^3$, periodic in u with period 2π, which represents a complete surface with negative Gauss curvature in $\mathbb{R}^3$? If it is not compact, can you prove its total curvature $\iint \kappa dA \div 2\pi$ is equal to its Euler characteristic χ? (25/100)