臺灣大學數學系

八十九學年度博士班入學考試題

幾何

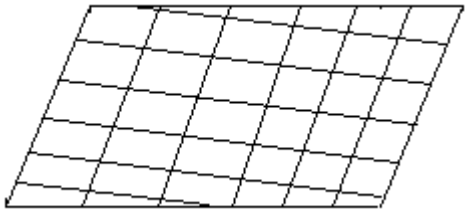
以下問答若未檢附論證或反例者視同瞎猜

1.

Can you find a 3-dimensional manifold so that its fundamental group π_1 is NOT isomorphic to its first homology group H_1? (25/100)

2.

曲面 M 若爲一族直線之聯集, 則稱 M 爲ruled surface,此時其高斯曲率 K 是否必爲零? ruled surface若同時又爲另一族直線之聯集, 則稱爲doubly ruled surface, 如下圖所示, doubly ruled surface是否必爲平面? (25/100)



3.

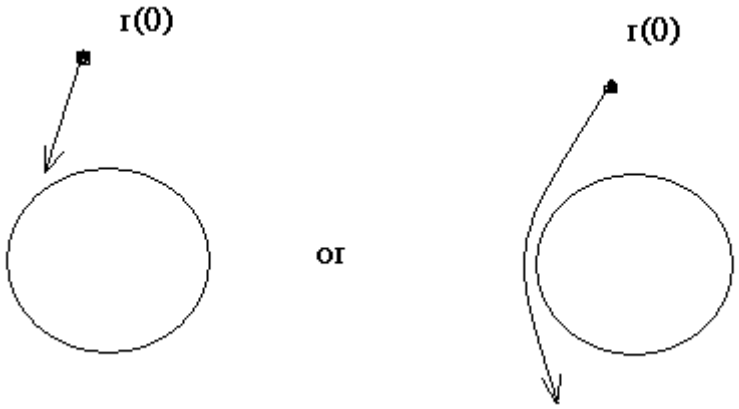
根據間諜衛星之相片, 欲繪製巨幅大陸地區之平面圖, 務使所有鐵公路均按 10^{-6} 比例尺確實標明, 令 θ, φ 爲地球之經緯度, x, y 爲平面圖之座標, 則

$x = x(\theta, \varphi) = ?, y = y(\theta, \varphi) = ?$ (25/100)

4.

$x^2 + y^2 > 1$, Riemannian metric

$$ds^2 = dx^2 + dy^2 + \frac{1}{x^2 + y^2 - 1} \cdot \frac{1}{x^2 + y^2} \cdot (x^2 dx^2 + 2xy dx dy + y^2 dy^2)$$



geodesic $r(t) = (x(t), y(t))$ has initial values

$r(0) = (0, \cosh 1) = (0, \frac{1}{2}(e + \frac{1}{e})), r'(0) = (-1, -1)$. Does $r(t)$ reach the boundary $x^2 + y^2 = 1$ at certain time $t > 0$? (25/100)