臺灣大學數學系

九十五學年度博士班入學考試題

代數

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1. Let $\phi(n)$ be the Euler totient function of a positive integer n, that is, the number of integers k such that $1 \leq k \leq n$ and $(k, n) = 1$.
   (a) Solve $\phi(n) = 12$.
   (b) Show that $\lim_{n\to\infty} \phi(n) = \infty$

2. Let G be a group of order pqr, where p, q, r are primes, not necessarily distinct. Show that G is not a simple group.

3. Let $\phi : A \to B$ be an additive mapping of a ring A into a ring B such that, for each pair $x, y \in A$, either $\phi(xy) = \phi(x)\phi(y)$ or $\phi(xy) = \phi(y)\phi(x)$. Show that ϕ is either a homomorphism or anti-homomorphism.

4. Determine whether the equations (a) $x^5+x^2-x+1 = 0$, (b) $x^5-4x+2 = 0$ are solvable in radicals over $\mathbb{Q}$.

5. Let $g, f_1, \cdots, f_r$ be linear functionals on a vector space V with respective null spaces $N, N_1, \cdots, N_r$. Show that g is a linear combination of $f_1, \cdots, f_n$ if and only if N contains the intersection $N_1 \cap \cdots \cap N_r$.