臺灣大學數學系

九十一學年度博士班入學考試題

代數

1.

Let N, H, K be normal subgroups of a group G.

(a)

Suppose $NK = HK$ and $N \cap K = H \cap K$. Is $N = H$ necessarily true?

(b)

Does G necessarily contain a subgroup isomorphic to G/N ?

(c)

Suppose $G/N \cong G/H$. Is $N \cong H$ necessarily true?

(d)

Suppose $G/N \cong G/H$ and $N \subseteq H$. Is $N = H$ necessarily true?

2.

Let α be a root of $x^3 - 3x + 4 = 0$ and $\beta = \alpha^2 + \alpha + 1 \in \mathbb{Q}(\alpha)$. Find the minimal polynomial of β over $\mathbb{Q}$ and express β^{-1} as $a\alpha^2 + b\alpha + c, a, b, c \in \mathbb{Q}$.

3.

Classify all prime ideals in $\mathbb{Z}[x]$.

4.

Let k be a field, and m, n be integers. Describe the followings as explicitly as possible:

(a)

$\text{Hom}_{\mathbb{Z}}(\mathbb{Z}/(n), \mathbb{Z}/(m))$.

(b)

$k[x]/(x^n) \otimes_{k[x]} k[x]/(x^m)$.

(c)

$k(x) \otimes_{k[x]} k(x)$.

5.

Let k be a field and $A \in M_{m \times n}(k), B \in M_{n \times p}(k)$. Show that AB can be written as a sum of n matrices of rank one.

6.

Find $A, B \in M_{2\times 2}(\mathbb{R})$ such that $e^A e^B \neq e^{A+B}$.