臺灣大學應用數學科學研究所 110 學年度碩士班甄試試題

科目：機率統計

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1. (4%) (6%) In the AIDS study, let D and T stand for the events of HIV+ patients and patients who are diagnosed as HIV+ patients, respectively. Suppose that the positive predictivity $P(D|T)$ and the negative predictivity $P(D^c|T^c)$ are known. What conditions are needed to obtain the sensitivity $P(T|D)$ and the specificity $P(T^c|D^c)$? How to compute the sensitivity and the specificity based on $P(D|T)$ and $P(D^c|T^c)$?

2. (10%) Let $X_1, \ldots, X_n$ be a random sample from a Poisson distribution with mean λ. Derive the conditional distribution of $X_1, \cdots, X_n$ on $\sum_{i=1}^n X_i$.

3. (5%) (10%) Let $X_1, \cdots, X_n$ be a random sample with the density function

$$f(x|\theta_0) = \exp(-(x - \theta_0))I(x \geq \theta_0).$$

Find a moment estimator of θ_0 and compute the mean squared error of the proposed estimator.

4. Let $\{X_{11}, \ldots, X_{1n_1}\}, \ldots, \{X_{k1}, \ldots, X_{kn_k}\}$ be k independent random samples from $N(\mu_{01}, \sigma_0^2), \ldots, N(\mu_{0k}, \sigma_0^2)$, respectively, $k \geq 3$.

(4a) (8%) Derive the the maximum likelihood estimator, say $(\widehat{\mu}_1, \ldots, \widehat{\mu}_k, \widehat{\sigma}^2)$, of $(\mu_{01}, \ldots, \mu_{0k}, \sigma_0^2)$.

(4b) (10%) Derive the sampling distribution of $(\widehat{\mu}_1, \ldots, \widehat{\mu}_k, \widehat{\sigma}^2)$.

(4c) (10%) Construct an approximated $(1-\alpha)$, $0 < \alpha < 1$, simultaneous Bonferroni confidence intervals for all $\mu_i - \mu_j$, $1 \leq i < j \leq k$.

5. Let $X_1, \ldots, X_n$ be a random sample from a $Bernoulli(p)$, $0 < p < 1$.

(5a) (10%) State the likelihood ratio testing procedure for the null hypothesis $H_0 : p = p_0$ versus the alternative hypothesis $H_A : p \neq p_0$, where p_0 is a known success probability.

(5b) (10%) Suppose that the sample size is large enough. Construct an approximated $(1-\alpha)$, $0 < \alpha < 1$, confidence interval for p.

6. (7%) (10%) Suppose that the test rejects the null hypothesis for large values of the test statistic T and that T has the continuous cumulative distribution function $F(\cdot)$ under the null hypothesis. Derive the distribution of $F(T)$ and establish the relation between the p-value V of the test and $F(T)$.