臺灣大學應用數學科學研究所 105 學年度碩士班甄試試題

科目：機率統計

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1. (8%) (7%) Let $f(t)$ and $F(t)$ stand for the respective probability density function and cumulative distribution function of a discrete non-negative random variable T with the support $\{t_1 < \ldots < t_m\}$, $S(t) = 1 - F(t)$, and $\lambda_i = P(T = t_i | T \geq t_i)$, $i = 1, \ldots, m$. Express $f(t_i)$ and $S(t_i)$ in terms of λ_i's.

2. (10%) Let X have a Gamma distribution with parameters $\alpha > 1$ and β. Compute the mean of the random quantity $1/X$.

3. (15%) Let $X_1, \ldots, X_n$ be a random sample from a continuous distribution $F(x)$ with the corresponding order statistics $X_{(1)}, \ldots, X_{(n)}$. Derive the distribution of $F(X_{(i)})$, $i = 1, \ldots, n$.

4. (10%) (15%) Let $X_1, \ldots, X_n$ be a random sample from $N(\mu, \sigma^2)$. Find the maximum likelihood estimator of $\Phi((x - \mu)/\sigma)$, where $\Phi(\cdot)$ represents the cumulative distribution function of a standard normal random variable and x is a given value, and derive its asymptotic distribution.

5. (8%) (7%) Let $X_1, \ldots, X_n$ be a random sample from $Poisson(\lambda)$ and λ have a $Gamma(\alpha, \beta)$ distribution. Find the posterior distribution of λ and the Bayes estimator of λ under the absolute error loss function.

6. Let $X_1, \ldots, X_n$ be a random sample from a density function $f(x|\lambda) = \theta e^{-\lambda x} I_{\{(0,\infty)\}}(x)$ with $\lambda > 0$.

(6a) (10%) Show that the rejection region of a likelihood ratio test of $H_0 : \lambda = \lambda_0$ versus $H_A : \lambda \neq \lambda_0$ is of the form $\{(X_1, \ldots, X_n) : \bar{X} e^{-\lambda_0 \bar{X}}\}$, where $\bar{X}$ is the sample mean of $X_1, \ldots, X_n$.

(6b) (10%) Find a valid p-value for the above hypotheses.