臺灣大學應用數學科學研究所109學年度碩士班甄試試題

科目：微分方程與線性代數

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1. Solve each of the following initial value problems. Determine the interval in which the solution is valid.

(a.) (20%)

$$\begin{cases} y' - \tan ty = e^t, \\ y(2) = 1. \end{cases}$$

(b.) (20%)

$$\begin{cases} y' = \frac{1}{x(y^2-1)}, \\ y(1) = -\frac{2}{3}. \end{cases}$$

2.(20%) Let

$$V_1 = (1,1,1,1),\ V_2 = (1,2,3,4),$$
$$V_3 = (4,3,2,1),\ V_4 = (1,4,2,3).$$

Are above vectors linearly independent? Find an orthonormal basis for $< V_1, V_2, V_3, V_4 >$.

3. (20%) Find the general solution to the system of differential equations

$$\begin{cases} x'(t) = x + y + z, \\ y'(t) = -3x + 4y + 2z, \\ z'(t) = 2x - y + z. \end{cases}$$

4. (20%) A and B are two symmetric real matrices. Show that they can be diagonalized simultaneously if and only if they commute, i.e.,

$$\exists O \text{ } s.t. \text{ } O^{-1}AO,\ O^{-1}BO \text{ are both diagonal matrices} \iff AB = BA.$$