臺灣大學應用數學科學研究所 107 學年度碩士班甄試試題

科目：微分方程與線性代數

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1. (30 pts) Let $(y(t), z(t))$ satisfy the system

$$\begin{cases} y''(t) + 11y(t) - 10z(t) = 1, \\ z'(t) + 6y(t) - az(t) = 0. \end{cases}$$

(a) Find the solution $(y(t), z(t))$ for $a = 0$.
(b) Find the solution $(y(t), z(t))$ for $a = 6$.

2. (40 pts) Let

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & -1 \\ 0 & 0.5 & 0 \\ 0 & -2 & 0.2 \end{pmatrix}, \ \mathbf{w}_0 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \text{ and } \mathbf{w}_k = \mathbf{A}^k \mathbf{w}_0 \text{ for } k = 1, 2, 3, \ldots$$

(a) Find $\mathbf{w}_k$ and $\lim_{k\to\infty} \mathbf{w}_k$.
(b) Solve $\mathbf{x}'(t) = \mathbf{A}\mathbf{x}(t)$.

3. (30 pts) Let $x(t)$ satisfy $x'(t) = g(t) - x(t)$.
(a) Assume $g(t) = \sin t$. Show that $\lim_{t\to\infty} x(t)$ does not exist.
(b) Assume $g(t) = \sin(t^3)$. Does $\lim_{t\to\infty} x(t)$ exist?