臺灣大學應用數學科學研究所 106 學年度碩士班甄試試題

科目：微分方程與線性代數

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1. Denote

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -4 & 0 & 5 & 0 \end{pmatrix}.$$

(A) (15 %) Find the inverse matrix of A.

(B) (10 %) Express the inverse matrix of A in terms of polynomial of A with degree less than 4.

(C) (10 %)Find all the eigenvalues of A.

(D) (25 %) Suppose $x(t)$ is the solution of the linear sysetm

$$x'(t) = Ax(t),$$

with the initial condition $x(0) = (2, 0, 2, 0)^T$. (Here, the notation T means transpose.). Find the exact value of $x(100)$.

2. (20 %) Solve the differential equation

$$\begin{cases} x''(t) - 2x'(t) + x(t) = e^t, \\ x'(0) = 1 \\ x(0) = 0. \end{cases}$$

3. (20 %) Let $A = (a_{ij})_{n\times n}$ be an n by n matrix defined as $a_{ij} = \frac{i}{j}$ for $i, j = 1, 2, \cdots, n$ and $n \geq 3$. Find all the eigenvalues and eigenvectors of A.