臺灣大學數學系

八十八學年度碩士班甄試入學考試試題

數理統計

1.

Suppose that $X_1, \ldots, X_n$ is an i.i.d. sample from the Uniform $(-\theta, \theta)$ distribution which has density $f(x) = 2^{-1}\theta^{-1}I_{(-\theta,\theta)}(x), \theta > 0$.

(a)

Find the maximum likelihood estimator of θ and denote it as $\hat{\theta}_1$.

(b)

Show that $\hat{\theta}_1$ is biased and has a simple rescaling, denote as $\hat{\theta}_2$, which is unbiased.

(c)

Find the variance of $\hat{\theta}_2$.

(d)

Find the limiting distribution function of the statistic $n(\theta - \hat{\theta}_2)$ as $n \to +\infty$.

2.

A company wants to estimate the proportion p, $0 < p < 1$, of defective items it produces. It is known that they rarely produce defective items. So n workers were asked to continue inspecting until they each has observed one defective item. Assume that the inspected items were selected randomly. Let X_i be the number of items the $i-$th worker inspected, $i = 1, \cdots, n$.

(a)

What statistic would you use to test the two guesses $p = 0.02$ and $p = 0.05$?

(b)

Describe a level-α test using the statistic you give in (a) and a good approximation to its distribution when $p = 0.02$ and n is large.