臺灣大學數學系

八十六學年度碩士班甄試入學考試試題

數理統計

1.

(25 points) Let X_{ij} $(i = 1, \ldots, p;\ j = 1, \ldots, k)$ be independent $N(\mu_i, \sigma^2)$ variables.

a.

(10 points) Show that the M.L.E.'s of $\mu_i,\ i = 1, \ldots, p$ and σ^2 are $\hat{\mu}_i = \frac{1}{k}\sum_{j=1}^{k} X_{ij}$ and $\hat{\sigma}^2 = \frac{1}{kp}\sum_{i=1}^{p}\sum_{j=1}^{k}(X_{ij} - \hat{\mu}_i)^2$.

b.

(10 points) Describe the distribution of $\hat{\mu}_i$ and the distribution of $\hat{\sigma}^2$.

c.

(10 points) Suppose k is fixed but $p \to \infty$. Is $\hat{\sigma}^2$ a good estimate of σ^2?

2.

(20 points) If $X_1, \ldots, X_n$ are iid $N(\theta, 1)$ random variables. The UMVU estimator of

$$p = P(X_1 \le 0)(= \Phi(-\theta))$$

is

$$\delta_n = \Phi\Big[-\bar{X}\sqrt{\frac{n}{n-1}}\Big],$$

where $\bar{X} = n^{-1}\sum_{i=1}^{n} X_i$ and $\Phi(\cdot)$ is the cumulative distribution function of standard normal random variable.

a.

(10 points) Suppose the assumption of normality is not valid but $E(X_i) = \theta$ and $Var(X_i) = 1$. If we continue to use δ_n as an estimator of $p = P(X_1 \le 0)$, describe the limiting behavior of δ_n? Give conditions under which δ_n is a consistent estimate of p?

b.

(10 points) Suppose the assumption of normality is not valid. Propose an alternative estimator of p and show that it is a consistent estimate of p.