臺灣大學數學系
九十八學年度碩士班甄試試題
科目：機率統計 2008.10.31

1. Let $U_1, \cdots, U_n$ and V be independent and identically distributed uniform random variables on the interval $[0, 1]$.

(1a) (10%) Find the joint probability density function of $U_{(1)} = \min\{U_1, \cdots, U_n\}$ and $U_{(n)} = \max\{U_1, \cdots, U_n\}$.

(1b) (10%) Find $P(U_{(1)} < V < U_{(n)})$.

2. (15%) Let X and Y have the joint probability density function $f(x, y) = e^{-y}$, $0 \le x \le y$. Find the joint probability density function of the random variables $E[X|Y]$ and $E[Y|X]$.

3. (15%) Let $X_1, \cdots, X_n$ be a random sample from a population $\{x_1, \cdots, x_N\}$. Find an unbiased estimator of $\sigma^2 = N^{-1} \sum_{i=1}^N (x_i - \mu)^2$, where $\mu = N^{-1} \sum_{i=1}^N x_i$.

4. (10%) (10%) Consider a random sample $X_1, \cdots, X_n$ from a population with density function

$$f(x|\mu, \sigma) = \frac{1}{2\sigma} e^{\frac{|x-\mu|}{\sigma}} 1_{(-\infty,\infty)}(x).$$

Find the maximum likelihood and moment estimators of μ and σ.

5. Let $X_i \sim Binomial(n_i, p_i)$, $i = 1, \cdots, m$, be independent.

(5a) (10%) Derive the likelihood ratio test for the null hypothesis $H_0 : p_1 = \cdots = p_m$ versus the alternative hypothesis $H_A : p_i \neq p_j$ for some $i \neq j$.

(5b) (10%) What is the large sample distribution of the test statistic?

6. (10%) Let X follow a Poisson distribution with rate λ. Find a variance-stabilizing transformation of X.