臺灣大學數學系

九十七學年度碩士班甄試試題

科目：機率統計 2007.11.02

1. (15%) Let T be a continuous failure time with the survival function $S(t) = P(T > t)$, $0 < t < \infty$, and $r(t) = E[T - t | T \geq t]$ be the expected residual life at time t. Show that $r(t) = \int_t^\infty S(u)du/S(t)$ provided that $E[T] < \infty$.

2. (15%) Let $T_1, \cdots, T_n$ be a random sample from an exponential distribution, and $T_{(1)}, \cdots, T_{(n)}$ be the corresponding order statistics. Derive the joint distribution of $U_1, \cdots, U_n$, where $U_i = (n - i + 1)(T_{(i)} - T_{(i-1)})$, $i = 1, \cdots, n$, with $T_{(0)} = 0$.

3. (15%) Suppose that $\widehat{\theta}$ is a (MVUE) minimum variance unbiased estimator of θ and let t be any estimator with $E[t] = 0$ and $Var(t) < \infty$. Show that $\widehat{\theta}$ and t are uncorrelated.

4. Let $X_1, \cdots, X_n$ be a random sample from a Poisson distribution with rate λ.

(4a) (15%) Find the uniformly minimum variance unbiased estimator of $\exp(-2\lambda)$.

(4b) (15%) Show that the power function of the uniformly most powerful level α test for $H_0 : \lambda \leq \lambda_0$ versus $H_A : \lambda > \lambda_0$ is increasing in λ.

5. (15%) Consider a simple linear regression model $Y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$, where ε_i's are uncorrelated with mean 0 and variance σ^2. Let $(\widehat{\beta}_0, \widehat{\beta}_1)$ be the unique least squares estimator of (β_0, β_1), $SST = \sum_{i=1}^n (Y_i - \bar{Y})^2$ and $SSR = \sum_{i=1}^n (\widehat{Y}_i - \bar{Y})^2$ with $\bar{Y} = \sum_{i=1}^n Y_i/n$ and $\widehat{Y}_i = \widehat{\beta}_0 + \widehat{\beta}_1 x_i$, and r be the sample correlation coefficient of $\{(x_i, Y_i)\}_{i=1}^n$. Show that $r^2 = SSR/SST$.

6. (10%) Show that $argmin_a \sum_{i=1}^n |Y_i - a| = M_Y$, where M_Y is the median of $Y_1, \cdots, Y_n$.