

臺灣大學數學系

101 學年度碩士班甄試試題

科目：機率統計

- (15%) Let X_1, \dots, X_n be a random sample from Bernoulli(p), where $n \geq 4$. Find the uniformly minimum variance unbiased estimator of p^4 .
- (15%) Let $I(f) = \int_a^b f(x)dx$ and X_1, \dots, X_n be a random sample from a density function $g(x)$ on $[a, b]$ with finite first moment. Show that $\hat{I}(f) = \frac{1}{n} \sum_{i=1}^n \{f(X_i)/g(X_i)\}$ converges in probability to $I(f)$ as $n \rightarrow \infty$.
- (15%) Let X be a binomial random variable with parameters n and $0 < p < 1$. Assume that $n \rightarrow \infty$, $p \rightarrow 0$, and $\lambda_n = np \rightarrow \lambda$ with $\lambda > 0$. Show that the moment generating function converges to the moment generating function of a Poisson random variable with parameter λ .
- (15%) Let X_1, \dots, X_n be a random sample from an exponential distribution with rate λ . What is the asymptotic variance of the maximum likelihood estimator of λ .
- Let X_1, \dots, X_n be a random sample from a uniform distribution on $[\theta_l, \theta_u]$.
 - (10%) Find the maximum likelihood estimators of θ_l and θ_u .
 - (10%) What is the joint distribution of the maximum likelihood estimators?
- (10%) (10%) Let X_1, \dots, X_n be a random sample from a Poisson distribution with rate λ . Derive the likelihood ratio test of $H_0 : \lambda = \lambda_0$ versus $H_A : \lambda > \lambda_0$ at level α , and show that the test is uniformly most powerful level α test.