臺灣大學數學系

100 學年度碩士班甄試試題

科目：機率統計

1. (10%) (5%) Let $W \sim N(\mu, \sigma^2)$ and $Y = \exp(W)$. Find the probability density function of Y and it's corresponding moments.

2. (10%) Let $X, Y, Z \overset{i.i.d}{\sim} N(0,1)$ with $(X,Y,Z) = (R\sin\Phi\cos\Theta, R\sin\Phi\sin\Theta, R\cos\phi)$, where the supports of R, Φ, and Θ are $(0,\infty)$, $(0,\pi)$, and $(0,2\pi)$, respectively. Find the joint distribution of (R,Φ,Θ).

3. (10%) Let $X_1, \cdots, X_n$ be a random sample with $E[h(X_1,X_2)] = \theta$, where $h(x,y)$ is a symmetric function. Moreover, let $X_{(1)}, \cdots, X_{(n)}$ denote the order statistics of $X_1, \cdots, X_n$. Derive the conditional expectation $E[h(X_1,X_2)|X_{(1)}, \cdots, X_{(n)}]$.

4 (8%) (7%) Let $I(f) = \int_a^b f(x)dx$ and $X_1, \cdots, X_n$ be a random sample from a density function $g(x)$ on $[a,b]$. Find an unbiased estimator of $I(f)$ and compute it's variance.

5. (10%) Find the minimizer of $S(a) = \sum_{i=1}^n |X_i - a| - 0.5(X_i - a)$.

6. (15%) Let $X_1, \cdots, X_{n+1}$ be a random sample from $Bernoulli(\pi)$ and $h(\pi) = P(\sum_{i=1}^n X_i > X_{n+1}|\pi)$. Find the uniformly minimum variance unbiased estimator of $h(\pi)$.

7. (10%) Let $X_1, \cdots, X_n$ be a random sample from a population with probability density function $f(x|\theta,\nu) = (\theta\nu^\theta/x^{\theta+1})1_{[\nu,\infty)}(x)$, where θ and ν are unknown positive parameters. Find the maximum likelihood estimators of θ and ν.

8. (15%) Let $X_1, \cdots, X_n$ be a random sample from $Uniform(\theta, \theta+1)$. Consider the hypotheses $H_0 : \theta = 0$ versus $H_A : \theta > 0$. Find the uniformly most powerful size α test.